Geometric Origin of Chemical Bonding in HPA–Ω: Deriving Molecular Stability from the Three-Channel Geometric Impedance α_{em} and the Internal Phase Volume $\mu = m_p/m_e$

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Abstract

Why does chemistry exist? In standard quantum chemistry, the existence and stability of chemical bonds depend crucially on two dimensionless inputs: the electromagnetic coupling strength (the fine-structure constant α_{em}) and the nuclear–electronic scale separation (the proton–electron mass ratio $\mu = m_p/m_e$). In the usual narrative, these appear as externally given parameters, so "chemical existence" is reduced to an accidental location of the universe in the $(\alpha_{\text{em}}, \mu)$-plane.

In the HPA–Ω framework (Holographic Polar Arithmetic / Ω-axiomatic program), we instead treat observable dimensionless constants as invariants of a scan–projection readout protocol. We adopt a strict layer discipline: a *closed-theory layer* that uses only explicit axioms and geometric definitions, and an *interface layer* that records the minimal matching needed to connect a closed model to operational constants. Within the closed layer, we recall two theorem-level geometric assignments established in the companion constants-geometry manuscript: a three-channel geometric impedance for electromagnetism,

$$\alpha_{\text{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi,$$

and an internal phase-volume cost for the proton–electron mass ratio,

$$\mu_{\text{geo}} = 6\pi^5.$$

We then show how these two geometric invariants control the two structural pillars of chemistry. First, α_{em} fixes the absolute chemical energy and length scales via atomic units ($a_0 \propto \alpha_{\text{em}}^{-1}$ and $E_h \propto \alpha_{\text{em}}^2$), explaining the eV-scale of chemistry as a rigid consequence of a dimensionless coupling. Second, μ fixes the Born–Oppenheimer small parameter $\varepsilon_{\text{BO}} \sim \mu^{-1/2}$ and hence the electronic/vibrational/rotational hierarchy and the stability of molecular geometry (including isotope effects). We further anchor these scale statements with rigorous stability-of-matter results (showing that α_{em} lies deep inside known stability regimes) and with the textbook H_2^+ Born–Oppenheimer curve as the minimal molecule-level reference.

Keywords: HPA–Ω; chemical bond; molecular stability; geometric impedance; fine-structure constant; proton–electron mass ratio; atomic units; Born–Oppenheimer; stability of matter.

Layer discipline. We keep a strict separation between: (i) a **closed-theory layer** (axioms/definitions ⇒ theorem-level invariants), (ii) an **interface layer** (minimal matching inputs that connect closed invariants to operational constants), and (iii) an **interpretation layer** (mapping to standard effective Hamiltonians and chemistry language). We treat log as the natural logarithm unless otherwise stated.

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Contents

1 From "parameter-input chemistry" to "geometric-inevitability chemistry"

1.1 The origin problem of chemistry

Chemistry is empirically robust: atoms and molecules exist stably across wide ranges of temperature and pressure, exhibit reproducible valence patterns, and possess sharp spectroscopic fingerprints. In the standard nonrelativistic account, chemical bonding emerges from the competition between electronic kinetic energy and Coulomb attraction, together with the Born–Oppenheimer (BO) separation that makes molecular geometry meaningful as a slowly varying nuclear scaffold [1, 2].

However, two dimensionless quantities dominate the very possibility of this structure:

- **Electromagnetic coupling strength** α_{em}, which fixes atomic-unit scales a_0 and E_h and hence the absolute energy/length scale of chemistry.
- **Nuclear–electronic mass hierarchy** $\mu = m_p/m_e$, which fixes the small parameter $\varepsilon_{\text{BO}} \sim \mu^{-1/2}$ and hence the separation between electronic, vibrational, and rotational degrees of freedom.

In the conventional narrative, α_{em} and μ are empirical inputs; chemical stability is then described as the fact that these happen to land in a "habitable window." This viewpoint makes the existence of chemistry appear as contingent.

1.2 HPA–Ω: constants as protocol invariants

The HPA–Ω program proposes a different starting point: observable dimensionless "constants" are not adjustable parameters but invariants of a scan–projection readout protocol. In this viewpoint, "probability," "noise," and "thermodynamics" are not external assumptions but emerge from finite-resolution readout. The framework therefore enforces a methodological discipline: identify a closed set of axioms and geometric definitions, derive invariants, then connect to operational constants through a minimal interface map [3–5].

Within this discipline, the constants-geometry manuscript [5] proposes and audits a concrete geometricization of α_{em} and μ:

$$\alpha_{\text{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi, \qquad \mu_{\text{geo}} = 6\pi^5. \tag{1}$$

The present paper asks a chemistry-facing question:

> Can chemical bonding and molecular stability be rephrased as necessary consequences of the same protocol-geometric invariants $(\alpha_{\text{em}}, \mu)$?

1.3 Contributions and structure

Our contributions are twofold.

1. We provide a chemistry-oriented bridge from the geometric invariants $(\alpha_{\text{geo}}, \mu_{\text{geo}})$ to atomic units and BO scale separation, isolating how they rigidly fix (i) the eV scale of chemistry and (ii) the stability of molecular geometry and isotope effects.
2. We anchor the above scale claims with rigorous stability-of-matter results (placing α_{em} inside known stability regimes) and with the textbook H_2^+ Born–Oppenheimer curve as the minimal molecule-level reference.

Section 2 recalls the minimal scan–readout interface and discrepancy language. Sections 3 and 4 record the closed-theory derivations of α_{geo} and μ_{geo} in a form tailored to chemical applications. Section 5 derives the atomic-unit scaling and the BO hierarchy. Section 6 records rigorous stability-window anchors. Section 7 uses H_2^+ as a textbook quantitative reference. Section 8 discusses interface matching and falsifiable routes. An optional mismatch-based interface template is recorded in Appendix B.

2 Minimal scan–readout interface and phase friction

This section recalls the minimal interface assumptions used throughout the HPA–Ω literature and fixes notation. We only require two axioms (readout-induced probability and a Weyl pair) plus a discrepancy definition used as a computable proxy for "phase friction" [3, 6–8].

2.1 O5: finite-resolution readout induces probability

Axiom 2.1 (O5: scan–projection readout induces probability)**.** *For a given readout resolution $\varepsilon > 0$, an observer has access to a POVM (or instrument) $\{E_k^{(\varepsilon)}\}_k$ on an effective observer Hilbert space $\mathcal{H}_{\text{eff}}$, satisfying $\sum_k E_k^{(\varepsilon)} = \mathbb{1}$. For an effective state (positive normalized functional) ω_{eff}, the outcome probabilities are*

$$P_k^{(\varepsilon)} = \omega_{\text{eff}}\left(E_k^{(\varepsilon)}\right). \tag{2}$$

This axiom encodes the HPA principle that probability is not an external sampling postulate but is induced by finite-resolution readout. For background on POVMs and instruments in standard quantum measurement theory, see [9]. In the standard density-operator representation, this reduces to $P_k^{(\varepsilon)} = \mathrm{Tr}(\rho_{\text{eff}}\, E_k^{(\varepsilon)})$.

2.2 O6: a Weyl pair and intrinsic noncommutativity

Axiom 2.2 (O6: Weyl pair)**.** *On $\mathcal{H}_{\text{eff}}$ there exist unitary operators U_{scan} and V satisfying the Weyl relation*

$$U_{\text{scan}} V = \mathrm{e}^{2\pi \mathrm{i}\nu} V U_{\text{scan}}, \qquad \nu \in (0,1) \setminus \mathbb{Q}. \tag{3}$$

In a standard model on $L^2([0,1))$, one may take $(U_{\text{scan}}\psi)(x) = \psi(x+\nu)$ and $(V\psi)(x) = \mathrm{e}^{2\pi \mathrm{i} x}\psi(x)$, so that the scan orbit is the Kronecker sequence $x_n = x_0 + n\nu \pmod 1$ [6]. For mathematical background on Weyl relations and phase-space representations, see [10].

2.3 Discrepancy and phase friction as mismatch accumulation

Finite readout resolution makes mismatch unavoidable. A convenient quantitative proxy is star discrepancy. For a point sequence $(x_1, \dots, x_N) \subset [0,1)$ define the empirical distribution function

$$F_N(a) = \frac{1}{N}\#\{1 \le n \le N :\ x_n < a\}, \qquad 0 \le a \le 1, \tag{4}$$

and the one-dimensional star discrepancy and its cumulative mismatch

$$D_N^* = \sup_{0 \le a \le 1} |F_N(a) - a|, \qquad E_N := N D_N^*. \tag{5}$$

The discrepancy literature provides sharp bounds for Kronecker sequences and related low-complexity protocols [7, 8].

Remark 2.3 (Higher-dimensional discrepancy)**.** *For multi-degree-of-freedom readouts, one uses multi-dimensional discrepancy notions (e.g. star discrepancy on $[0,1)^d$) and corresponding Koksma–Hlawka-type bounds; see [7, 8]. In this paper, we use the one-dimensional definition only as a minimal interface proxy and only in the optional appendix-level template (Appendix B).*

Remark 2.4 (Quantitative size for Kronecker scanning)**.** *For Kronecker sequences* $x_n = x_0 + n\nu \pmod 1$*, discrepancy bounds are controlled by the continued-fraction data of* ν*. In particular, for bounded partial quotients (e.g. the golden-ratio case relevant in the HPA literature), one has logarithmic discrepancy bounds of the form* $D_N^* = O((\log N)/N)$ *and hence* $E_N = O(\log N)$*; see [7, 8].*

In HPA phase thermodynamics, a computable "phase-friction entropy proxy" can be defined by

$$S_{\mathrm{pf}}(N) = k_B E_N, \tag{6}$$

so that mismatch accumulation plays the role of an irreducible cost under coarse-grained (unconditional) readout [11].

For chemistry, this language will be used only in the interpretation layer: bonding and reactions can be modeled as rearrangements that reduce mismatch accumulation under a given readout interface (Appendix B).

3 The geometric origin of α_{em}: three-channel impedance

Chemistry requires an absolute electromagnetic energy scale. In standard physics this scale is set by the dimensionless electromagnetic invariant α_{em} through atomic units. In the HPA–Ω constant-geometry program, α_{em} is a *target* dimensionless invariant of the readout protocol, not an external knob; the minimal closed model produces a theorem-level geometric prediction α_{geo} for its value. We record the minimal closed-theory chain that yields the theorem-level value

$$\alpha_{\mathrm{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi, \tag{7}$$

following the constants-geometry manuscript [5].

3.1 Serial composition and logarithmic readout cost

Axiom 3.1 (Serial composition and geometric impedance)**.** *If a readout protocol must satisfy constraints in a fixed hierarchy of channels, assign each channel* j *a multiplicative weight* $w_j \in (0, 1]$ *and define its readout cost by* $V_j := -\log w_j$*. The total weight is*

$$w_{\mathrm{tot}} = \prod_j w_j, \tag{8}$$

and the associated geometric impedance is the additive log-cost

$$\alpha_{\mathrm{geo}}^{-1} := -\log w_{\mathrm{tot}} = \sum_j V_j. \tag{9}$$

Remark 3.2. *The axiom enforces a precise meaning of "impedance": the ontological composition is multiplicative; additivity is an induced feature after the logarithmic readout projection. This logarithmic additivity is the standard "product-to-sum" mechanism underlying information measures and cost aggregation; see, e.g., [12–14].*

3.2 Three strata and canonical phase volumes

Axiom 3.3 (Geometric cost as canonical volume)**.** *For each channel* j*, the process cost is identified with a canonical geometric invariant of its compact phase space* $\mathcal{M}_j$*. In the minimal model,*

$$V_j = \mathrm{Vol}(\mathcal{M}_j), \tag{10}$$

where Vol *is the volume induced by the standard bi-invariant metric on compact Lie groups and quotient metrics.*

Axiom 3.4 (Electromagnetic three-stratum phase spaces)**.** *The electromagnetic readout channel is represented by three topologically distinct strata:*

$$\mathcal{M}_{\text{bulk}} \cong U(1) \times SU(2), \qquad \mathcal{M}_{\text{boundary}} \cong SO(3), \qquad \mathcal{M}_{\text{line}} \cong \mathbb{R}P^1. \tag{11}$$

Remark 3.5. *The projective quotients encode the intrinsic ray identification of readout. In particular, $SU(2) \to SO(3) = SU(2)/\{\pm 1\}$ reflects the spinorial double cover, and $\mathbb{R}P^1 = U(1)/\{\pm 1\}$ reflects phase-ray identification [15, 16].*

3.3 Theorem-level value from group volumes

Theorem 3.6 (Three-channel geometric impedance)**.** *Under Axioms 3.1–3.4, the minimal-model geometric value of the inverse fine-structure constant is*

$$\alpha_{\text{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.0363037759. \tag{12}$$

Proof. By Axiom 3.1, $\alpha_{\text{geo}}^{-1} = V_{\text{bulk}} + V_{\text{boundary}} + V_{\text{line}}$. By Axiom 3.3, $V_j = \mathrm{Vol}(\mathcal{M}_j)$, and by Axiom 3.4 this reduces to canonical volumes. Standard values [15, 16] are

$$\mathrm{Vol}(U(1)) = 2\pi, \quad \mathrm{Vol}(SU(2)) = 2\pi^2, \quad \mathrm{Vol}(SO(3)) = \pi^2, \quad \mathrm{Vol}(\mathbb{R}P^1) = \pi. \tag{13}$$

Therefore

$$V_{\text{bulk}} = \mathrm{Vol}(U(1) \times SU(2)) = 4\pi^3, \qquad V_{\text{boundary}} = \pi^2, \qquad V_{\text{line}} = \pi, \tag{14}$$

and the sum gives $4\pi^3 + \pi^2 + \pi$. ☐

3.4 Interface matching to CODATA

Let α_{exp} denote the CODATA recommended fine-structure constant [17]. Define the inverse-impedance gap

$$\Delta\alpha^{-1} := \alpha_{\text{geo}}^{-1} - \alpha_{\text{exp}}^{-1}. \tag{15}$$

In the multiplicative readout variable $w = \exp(-\alpha^{-1})$, the mismatch is absorbed by a single matching input

$$s_\alpha := \frac{w_{\text{exp}}}{w_{\text{geo}}} = \exp(\Delta\alpha^{-1}), \tag{16}$$

which encodes the interface between the minimal geometric model and operational low-energy coupling [5]. In this paper we keep the interface explicit rather than treating it as a tuning parameter: s_α is recorded and can be propagated into any chemistry-facing prediction requiring high precision.

Remark 3.7 (Why the weight variable $w = \exp(-\alpha^{-1})$)**.** *The variable $w = \exp(-\alpha^{-1})$ is the natural multiplicative counterpart of the additive impedance α^{-1} under the logarithmic readout map. It is introduced only to make the interface matching multiplicative (a single factor s_α), avoiding hidden additive retuning across different applications.*

Quantitative size. Using the CODATA 2022 central inverse coupling quoted in [5, 17], one has

$$\Delta\alpha^{-1} \approx 3.046 \times 10^{-4}, \qquad \frac{\Delta\alpha^{-1}}{\alpha_{\text{exp}}^{-1}} \approx 2.22 \times 10^{-6}, \qquad s_\alpha \approx 1.0003046.$$

Thus the interface mismatch is at the few-ppm level in inverse impedance and at the few-ppm level in any derived chemical scale that depends smoothly on α_{em}.

3.5 Rigidity at bounded complexity (no coefficient tuning)

A key concern for any constant-geometry claim is coefficient tuning. The constants-geometry manuscript records an explicit bounded-complexity rigidity check: minimize the CODATA error within the ansatz $a\pi^3 + b\pi^2 + c\pi$ over small nonnegative integers. We quote the result as an internal rigidity anchor.

Proposition 3.8 (Uniqueness at low coefficient complexity)**.** *Within the coefficient-sum complexity domain* $a, b, c \in \mathbb{Z}_{\geq 0}$ *and* $a + b + c \leq 10$*, the unique minimizer of* $|a\pi^3 + b\pi^2 + c\pi - \alpha_{\text{exp}}^{-1}|$ *is* $(a, b, c) = (4, 1, 1)$*, with relative error* $\sim 2.2 \times 10^{-6}$*. The next-best triple in the same domain has relative error at least* 3.2×10^{-3}*.*

Proof. This is a finite exhaustive enumeration recorded in [5]. □

4 The geometric origin of $\mu = m_p/m_e$: internal phase volume and scale separation

Chemistry requires not only bound electronic states but also a robust separation between fast electronic and slow nuclear motion. This separation is controlled by the dimensionless proton–electron mass ratio $\mu = m_p/m_e$. In the HPA–Ω constant-geometry program, μ is again a *target* dimensionless invariant of internal protocol geometry; the minimal closed model yields the phase-volume prediction μ_{geo}. We record the minimal closed-theory chain yielding

$$\mu_{\text{geo}} = 6\pi^5, \tag{17}$$

following [5].

4.1 Mass ratios as phase-volume costs (closed layer)

Definition 4.1 (Mass-ratio invariants)**.** *For particle classes* i, j*, define the dimensionless ratio*

$$\frac{m_i}{m_j} = \frac{I_i}{I_j}, \tag{18}$$

where I_i *is a protocol-geometric invariant induced by the internal defect/holonomy data of class* i*.*

Axiom 4.2 (Mass invariants as phase-volume costs)**.** *For each particle class* i*, the invariant* I_i *is given by a canonical phase volume of an internal compact manifold* $\mathcal{M}_i$ *(or a finite disjoint union), normalized so that the electron satisfies* $I_e = 1$*:*

$$I_i = \text{Vol}(\mathcal{M}_i), \qquad \text{Vol}\left(\bigsqcup_{a=1}^{A} \mathcal{M}_{i,a}\right) = \sum_{a=1}^{A} \text{Vol}(\mathcal{M}_{i,a}). \tag{19}$$

4.2 Proton internal phase space (three-color sector sum)

Axiom 4.3 (Proton internal phase space)**.** *Model the proton as a sum over three identical color sectors* ($N_c = 3$) *[18]. Each color sector is assigned the compact phase space*

$$\mathcal{M}_q \cong SO(3) \times SO(3) \times U(1), \tag{20}$$

so that the proton internal space $\mathcal{M}_p$ *is the disjoint union of three copies of* $\mathcal{M}_q$*.*

Remark 4.4. *At the level of canonical volumes, this choice is equivalent to* $SO(3) \times SU(2) \times \mathbb{R}P^1$ *because* $\text{Vol}(SO(3))\text{Vol}(U(1)) = \text{Vol}(SU(2))\text{Vol}(\mathbb{R}P^1)$*. The convention above treats rotational sectors projectively while keeping the phase circle unquotiented [5].*

4.3 Theorem-level value and interface matching

Theorem 4.5 (Proton–electron mass ratio)**.** *Under Axioms 4.2–4.3, the geometric prediction for the proton–electron mass ratio is*

$$\mu_{\mathrm{geo}} := \frac{m_p}{m_e} = 6\pi^5 \approx 1836.1181087117. \tag{21}$$

Proof. By Definition 4.1 and Axiom 4.2, $m_p/m_e = I_p/I_e = I_p$ since $I_e = 1$. By Axiom 4.3,

$$I_p = 3\,\mathrm{Vol}(SO(3) \times SO(3) \times U(1)). \tag{22}$$

Using $\mathrm{Vol}(SO(3)) = \pi^2$ and $\mathrm{Vol}(U(1)) = 2\pi$ yields

$$I_p = 3(\pi^2)(\pi^2)(2\pi) = 6\pi^5. \tag{23}$$

☐

Let μ_{exp} denote the CODATA recommended value [17]. Define the multiplicative interface matching input

$$s_\mu := \frac{\mu_{\mathrm{exp}}}{\mu_{\mathrm{geo}}}. \tag{24}$$

As emphasized in [5], s_μ is not a per-system tuning knob but a single interface encoding non-perturbative QCD binding and radiative structure. In chemistry-facing applications, the key output is not the exact value of μ itself but the existence of a large hierarchy $\mu \gg 1$ and the induced expansion parameter $\mu^{-1/2}$.

Quantitative size. Using the CODATA 2022 central value quoted in [5, 17], the offset is

$$\Delta\mu := \mu_{\mathrm{geo}} - \mu_{\mathrm{exp}} \approx -3.456 \times 10^{-2}, \qquad \frac{\Delta\mu}{\mu_{\mathrm{exp}}} \approx -1.88 \times 10^{-5}, \qquad s_\mu \approx 1.0000188.$$

4.4 Rigidity signals (no tuning): factorization and bounded complexity

The constants-geometry manuscript records two rigidity checks relevant for μ.

Proposition 4.6 (Finite primitive factorization rigidity)**.** *Fix the primitive candidate set* $\{U(1), SU(2), SO(3),$ *with canonical volumes as in [5]. Restrict a per-color sector to be a product of three primitives (order irrelevant). Then the condition* $3\,\mathrm{Vol}(\mathcal{M}_q) = 6\pi^5$ *holds if and only if*

$$\mathcal{M}_q \cong SO(3) \times SO(3) \times U(1) \qquad \text{or} \qquad \mathcal{M}_q \cong SO(3) \times SU(2) \times \mathbb{R}P^1.$$

In particular, among the 20 *three-factor multisets, these two are the unique solutions; the next-closest candidate has order-one relative error.*

Proof. This is a finite enumeration recorded in [5]. ☐

Proposition 4.7 (Uniqueness at low coefficient complexity)**.** *Within the coefficient-sum complexity domain* $a, b, c, d, e \in \mathbb{Z}_{\geq 0}$ *and* $a+b+c+d+e \leq 10$*, the unique minimizer of* $|a\pi^5 + b\pi^4 + c\pi^3 + d\pi^2 + e\pi - \mu_{\mathrm{exp}}|$ *is* $6\pi^5$*, with relative error* $\sim 1.9 \times 10^{-5}$*. The next-best combination in the same domain has relative error at least* 4.4×10^{-4}*.*

Proof. This is a finite exhaustive enumeration recorded in [5]. ☐

5 From $(\alpha_{\rm em}, \mu)$ to chemistry: atomic units and Born–Oppenheimer hierarchy

This section isolates a key structural fact: in a nonrelativistic Coulombic molecule, $\alpha_{\rm em}$ fixes the absolute units (Hartree and Bohr) while μ fixes the small parameter controlling nuclear motion. Consequently, once $(\alpha_{\rm em}, \mu)$ are treated as rigid protocol invariants rather than empirical inputs, two necessary prerequisites of chemistry—an absolute eV-scale and a robust BO hierarchy—become rigid consequences of protocol geometry (with stability anchored separately in Section 6).

5.1 Atomic units: why chemistry lives at the eV scale

In standard physics, atomic units are defined by the Bohr radius and Hartree energy [17]:

$$a_0 = \frac{\hbar}{m_e c\, \alpha_{\rm em}}, \qquad E_h = m_e c^2\, \alpha_{\rm em}^2, \qquad {\rm Ry} = \frac{1}{2} E_h. \tag{25}$$

Numerically, $a_0 \approx 0.529\,\text{Å}$ and $E_h \approx 27.21\,\text{eV}$ [17]. In SI units, $m_e c^2$ is a MeV-scale quantity, and E_h is suppressed by $\alpha_{\rm em}^2 \sim 10^{-4}$, landing at the characteristic chemical scale of tens of eV. Typical bond energies are fractions of a Hartree, so eV-scale chemistry is a direct consequence of a dimensionless weak coupling. See, e.g., standard molecular quantum mechanics discussions in [19].

In the HPA–Ω constants program, $\alpha_{\rm em}$ is not a free input: the minimal closed-theory model predicts $\alpha_{\rm geo}^{-1} = 4\pi^3 + \pi^2 + \pi$ (Theorem 3.6). Hence the absolute chemical energy scale is locked by protocol geometry, up to the explicit interface matching s_α.

5.2 Dimensionless molecular Hamiltonian in atomic units (model assumptions)

Consider a molecule with N electrons (mass m_e) and nuclei labeled by A with charges Z_A and masses M_A. In atomic units (length in a_0, energy in E_h), the nonrelativistic Coulomb Hamiltonian takes the dimensionless form [2]

$$\frac{H}{E_h} = -\frac{1}{2}\sum_{i=1}^{N} \nabla_i'^2 - \frac{1}{2}\sum_{A} \frac{1}{\mu_A}\nabla_A'^2 - \sum_{i,A} \frac{Z_A}{r'_{iA}} + \sum_{i<j} \frac{1}{r'_{ij}} + \sum_{A<B} \frac{Z_A Z_B}{R'_{AB}}, \tag{26}$$

where $\mu_A := M_A/m_e$ and primes denote coordinates in units of a_0. This is the standard nonrelativistic point-charge Coulomb model used as the baseline of quantum chemistry; relativistic, QED, and finite-nuclear-size effects are treated as corrections at higher precision [2, 19].

Two points are structurally decisive:

- $\alpha_{\rm em}$ no longer appears in the *dimensionless* Hamiltonian (26); it only survives through the choice of absolute units a_0 and E_h.
- All nuclear slowness enters through the large parameters μ_A (in particular μ for hydrogenic nuclei).

Thus $\alpha_{\rm em}$ fixes the overall absolute scale of chemistry, while μ fixes the expansion parameter governing nuclear dynamics.

5.3 Born–Oppenheimer parameter and spectral hierarchy

For diatomic molecules with identical nuclei of mass $M \sim m_p$, a standard BO small parameter is

$$\varepsilon_{\rm BO} \sim \sqrt{\frac{m_e}{M}} \sim \mu^{-1/2}. \tag{27}$$

This parameter controls (i) nonadiabatic couplings between electronic surfaces and (ii) the hierarchy of energy scales. Dimensionally, one obtains the standard estimates

$$E_{\text{elec}} \sim E_h, \qquad E_{\text{vib}} \sim \frac{E_h}{\sqrt{\mu}}, \qquad E_{\text{rot}} \sim \frac{E_h}{\mu}. \tag{28}$$

For $\mu \simeq 1836$, one has $\mu^{-1/2} \simeq 0.023$, consistent with the robust empirical hierarchy (electronic $\gg$ vibrational $\gg$ rotational). See standard molecular quantum mechanics texts for these scalings and their refinements [2, 19].

5.4 A stability criterion: zero-point motion versus well depth

Molecular "geometry" is meaningful only if nuclear zero-point motion does not wash out the electronic potential well. Let D_e denote a typical electronic well depth (in Hartree units) and approximate the vibrational frequency near equilibrium by $\omega \sim \sqrt{k/\mu_{\text{red}}}$, where μ_{red} is a nuclear reduced mass in electron-mass units and k is the dimensionless curvature of the BO curve. Then the vibrational zero-point energy scales as

$$E_{\text{ZPE}} \sim \frac{1}{2}\omega \propto \mu^{-1/2}. \tag{29}$$

A minimal stability requirement is $E_{\text{ZPE}} \ll D_e$, which becomes a scale inequality of the form

$$\mu^{1/2} \gg \frac{1}{\eta}, \qquad D_e \sim \eta\, E_h, \quad \eta \in (10^{-2}, 10^{-1}) \text{ (typical)}. \tag{30}$$

With $\mu^{1/2} \sim 43$, the inequality is naturally satisfied for a wide range of chemical η, explaining why stable molecular geometries are abundant. The order-of-magnitude range for η is consistent with textbook bond-energy scales in atomic units [19].

5.5 Isotope effects as μ-controlled interface

Replacing a nucleus by an isotope changes M and hence μ_A while leaving α_{em} unchanged. Consequently vibrational frequencies and zero-point energies shift as $M^{-1/2}$, producing isotopic shifts in spectra and kinetic isotope effects. In the HPA–Ω viewpoint, this is not an empirical coincidence: isotope effects are direct consequences of the same internal phase-volume invariant μ that enforces the BO hierarchy.

6 A rigorous stability window: why chemistry is not destroyed by collapse

Sections 3–5 showed that $(\alpha_{\text{em}}, \mu)$ rigidly fix the absolute atomic-unit scale and the BO hierarchy. A reviewer will rightly ask a sharper question:

> Do these inputs merely set *scales*, or do they also ensure *stability* (no catastrophic collapse) in the underlying Coulombic matter model?

This section records the relevant rigorous facts from mathematical physics. We do not reprove them; they are mature results and are used as external anchors.

6.1 Nonrelativistic Coulomb matter is stable (bounded below)

Consider the standard nonrelativistic many-body Coulomb Hamiltonian for electrons and nuclei (with fixed nuclear charges and masses). The *stability of matter* problem asks whether the ground-state energy is bounded below linearly in particle number (so that macroscopic matter does not collapse as $N \to \infty$). This is a classic theorem: stability holds for nonrelativistic Coulomb matter, established in foundational works and summarized in modern expositions [20–23].

For the present paper, the consequence is simple: in the standard nonrelativistic Coulomb model that underlies quantum chemistry, the Hamiltonian is well-defined and extensive. Therefore, once the absolute scale is fixed by α_{em} and the nuclear slowness by μ, there is no hidden "instability loophole" that would forbid chemistry.

Minimal assumptions (external theorem input). The stability-of-matter theorems are statements about the standard quantum many-body Coulomb model under physically standard hypotheses: fermionic electrons (Pauli principle), Coulomb interactions, and (for thermodynamic stability statements) appropriate charge constraints and/or neutrality conditions. Precise formulations and assumptions are given in the cited mathematical physics literature [23].

Quantitative form (stability of the second kind). One standard formulation is that the ground-state energy $E_0(N, K)$ of a Coulomb system with N electrons and K nuclei admits a linear lower bound

$$E_0(N, K) \geq -C\,(N + K), \tag{31}$$

for a constant C independent of N, K (under the stated hypotheses). This is the mathematical statement that macroscopic matter does not collapse as particle number grows [23].

6.2 Relativistic constraints give an α-window

When relativistic kinematics are included (Dirac-type operators and/or magnetic fields), stability becomes nontrivial and depends on α_{em} and nuclear charges Z_A. There are sharp one-particle thresholds and distinguished self-adjointness regimes for Dirac–Coulomb operators, as well as sufficient conditions for many-body stability; see [23–25] for detailed statements and references. A widely used stability indicator is the requirement that $Z\alpha_{\text{em}}$ remain below the critical $O(1)$ thresholds associated with Dirac–Coulomb collapse (point-nucleus heuristics often quote $Z\alpha_{\text{em}} < 1$ as the basic bound-state regime).

In particular, $\alpha_{\text{em}} \approx 1/137$ lies deep in the weak-coupling regime relative to the known relativistic thresholds (which are $O(1)$), and therefore does not trigger relativistic collapse for the range of nuclear charges realized in chemistry. The inequality $Z\alpha_{\text{em}} < 1$ is used here only as an intuitive window marker; rigorous statements depend on the precise operator model (point nucleus vs. extended charge, single-particle vs. many-body, projection choices) and are detailed in the cited references.

A concrete numerical bound for the periodic table. For the largest nuclear charges realized in nature ($Z \leq 118$), one has

$$Z\,\alpha_{\text{em}} \lesssim \frac{118}{137} \approx 0.861,$$

which lies below the standard point-nucleus Dirac–Coulomb bound-state threshold $Z\alpha_{\text{em}} < 1$ and (numerically) even below the classical essential self-adjointness window quoted in the mathematical literature (see, e.g., [24] for precise statements). This makes the "relativistic collapse" concern quantitatively irrelevant for chemistry at observed Z.

6.3 Closed-theory implication: α_{geo} and μ_{geo} land inside the window

In the HPA–Ω constants program, the minimal closed-theory model fixes

$$\alpha_{\text{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi, \qquad \mu_{\text{geo}} = 6\pi^5.$$

Numerically, $\alpha_{\rm geo} \simeq 7.297 \times 10^{-3}$ and $\mu_{\rm geo} \simeq 1.836 \times 10^{3}$ [5]. Two derived quantities control chemistry directly:

$$\frac{E_h}{m_e c^2} = \alpha_{\rm em}^2, \qquad \varepsilon_{\rm BO} \sim \mu^{-1/2}. \tag{32}$$

Thus the closed-theory prediction implies

$$\alpha_{\rm geo}^2 \simeq 5.33 \times 10^{-5}, \qquad \mu_{\rm geo}^{-1/2} \simeq 2.33 \times 10^{-2},$$

which simultaneously (i) pushes the chemical scale down from MeV to eV and (ii) enforces a robust electronic/vibrational/rotational hierarchy. Moreover, the small interface mismatches between $(\alpha_{\rm geo}, \mu_{\rm geo})$ and operational constants are at the 10^{-6}–10^{-5} level [5,17]; their propagated effect on chemical scales is correspondingly tiny.

Propagation to chemical scales. At the level of atomic units,

$$a_0 \propto \alpha_{\rm em}^{-1}, \qquad E_h \propto \alpha_{\rm em}^2, \qquad \varepsilon_{\rm BO} \sim \mu^{-1/2}.$$

Therefore a relative perturbation $\delta\alpha/\alpha$ induces

$$\frac{\delta a_0}{a_0} = -\frac{\delta\alpha}{\alpha}, \qquad \frac{\delta E_h}{E_h} = 2\,\frac{\delta\alpha}{\alpha},$$

while a relative perturbation $\delta\mu/\mu$ induces

$$\frac{\delta(\mu^{-1/2})}{\mu^{-1/2}} = -\frac{1}{2}\,\frac{\delta\mu}{\mu}.$$

With the interface sizes quoted in [5], the propagated effects remain at the few-ppm level for (a_0, E_h) and at the $\sim 10^{-5}$ level for $\mu^{-1/2}$.

7 A textbook anchor: H_2^+ and the (α, μ) control of scale and hierarchy

To keep the chemistry discussion anchored to a concrete system without introducing unnecessary numerical artifacts, we use the simplest bound molecule as a textbook reference: the hydrogen molecular ion H_2^+ (two protons and one electron). Its BO potential curve is a classical worked example in molecular quantum mechanics and can be treated analytically (or numerically to essentially arbitrary precision) in standard references [2, 19, 26]. Our purpose here is not to improve accuracy, but to isolate two structural consequences of $(\alpha_{\rm em}, \mu)$ that are already visible in this minimal system:

- the **absolute scale** of $R_{\rm eq}$ and D_e is fixed once a_0 and E_h are fixed, hence by $\alpha_{\rm em}$;
- the **vibrational hierarchy** is controlled by $\mu^{-1/2}$ through nuclear reduced mass.

7.1 Born–Oppenheimer setup in atomic units

Fix the internuclear distance R (in Bohr units). The electronic Hamiltonian is

$$\hat{H}_e(R) = -\frac{1}{2}\nabla^2 - \frac{1}{r_A} - \frac{1}{r_B}, \tag{33}$$

with nuclei placed at $\mathbf{R}_A = (0, 0, -R/2)$ and $\mathbf{R}_B = (0, 0, R/2)$. Let $E_e(R)$ be the electronic ground-state energy. The BO potential curve for nuclear motion is

$$E(R) = E_e(R) + \frac{1}{R}. \tag{34}$$

Bonding corresponds to a local minimum of $E(R)$ below the dissociation limit $E(\infty) = -1/2$ (in Hartree).

7.2 Scale statement: dimensionless geometry is α-independent

In the nonrelativistic Coulomb model, after rescaling to atomic units, $\alpha_{\rm em}$ disappears from the dimensionless Hamiltonian (Section 5). Consequently, dimensionless molecular geometry (e.g. $R_{\rm eq}$ expressed in Bohr) is determined by the dimensionless Coulombic problem and does not depend on $\alpha_{\rm em}$. The *absolute* bond length is then

$$R_{\rm eq}^{(\rm SI)} = R_{\rm eq}^{(\rm Bohr)}\, a_0, \qquad a_0 = \frac{\hbar}{m_e c\, \alpha_{\rm em}},$$

so $\alpha_{\rm em}$ fixes the overall length scale. This statement is understood within the nonrelativistic Coulomb model; small relativistic and QED corrections reintroduce mild $\alpha_{\rm em}$-dependence at high precision [19].

7.3 Textbook quantitative anchor

For H_2^+, standard treatments find a bound minimum at an equilibrium separation $R_{\rm eq}$ of order $2\,a_0$, with a well depth of order $10^{-1}\,E_h$ (relative to the $\mathrm{H}+p$ limit) [19, 26]. Numerically, representative values are

$$R_{\rm eq} \approx 2.0\,a_0, \qquad E(R_{\rm eq}) \approx -0.60\,E_h, \qquad D_e := E(\infty) - E(R_{\rm eq}) \approx 0.10\,E_h,$$

consistent across standard references. These are exactly the kind of scale data emphasized here: order-unity dimensionless geometry in Bohr units and sub-Hartree well depth in Hartree units. Using $a_0 \approx 0.529\,\text{Å}$ and $E_h \approx 27.21\,\mathrm{eV}$ [17], this corresponds to

$$R_{\rm eq} \approx 1.06\,\text{Å}, \qquad D_e \approx 2.7\,\mathrm{eV},$$

illustrating the chemically natural length and energy scales as direct consequences of atomic units.

7.4 Nuclear zero-point motion and the μ control

Near $R_{\rm eq}$, approximate the BO curve by a harmonic potential:

$$E(R) \approx E(R_{\rm eq}) + \tfrac{1}{2}k(R - R_{\rm eq})^2, \qquad k = E''(R_{\rm eq}).$$

For two identical nuclei of mass ratio $\mu = M/m_e$, the reduced mass (in electron-mass units) is $\mu_{\rm red} = \mu/2$. The vibrational frequency in atomic units is

$$\omega \approx \sqrt{\frac{k}{\mu_{\rm red}}}, \tag{35}$$

and the zero-point energy is $E_{\rm ZPE} \approx \omega/2$. Therefore

$$E_{\rm ZPE} \propto \mu^{-1/2}, \tag{36}$$

which is the quantitative statement behind the qualitative BO "frozen geometry" picture. In particular, the large closed-theory value $\mu_{\rm geo} = 6\pi^5$ implies $\mu^{-1/2} \approx 2.3 \times 10^{-2}$ and therefore a strong separation between electronic and vibrational scales.

8 Interface corrections and falsifiability routes

The closed-theory values $\alpha_{\rm geo}$ and $\mu_{\rm geo}$ are not claimed to coincide identically with operational constants; instead, the HPA layer discipline requires that discrepancies be isolated as explicit interface inputs, not absorbed as hidden retuning. This section records the interface variables and outlines falsifiable spectroscopy routes.

8.1 Two small mismatches and two matching inputs

For α_{em}, define the inverse-impedance gap $\Delta\alpha^{-1} = \alpha_{\mathrm{geo}}^{-1} - \alpha_{\mathrm{exp}}^{-1}$ and the multiplicative matching input $s_\alpha = \exp(\Delta\alpha^{-1})$ (Section 3). For μ, define $s_\mu = \mu_{\mathrm{exp}}/\mu_{\mathrm{geo}}$ (Section 4). Numerically, the relative gaps are at the 10^{-6}–10^{-5} level [5, 17].

In chemistry-facing applications, these matching inputs propagate into:

- absolute unit conversions (energies in eV, lengths in Angstrom) through E_h and a_0 (controlled by s_α),
- nuclear kinematics through μ (controlled by s_μ),
- any additional interface corrections through explicit, low-parameter matching inputs (e.g. resolution parameters) rather than hidden per-system retuning.

8.2 Precision spectroscopy as a falsifiability channel

Beyond the scale-setting claims of the main text, one may ask whether finite-resolution readout consistency leaves additional, structured corrections to molecular energies. A mismatch-augmented interface template is recorded in Appendix B. If such corrections are physically present, they must leave residual signatures in systems where standard theory is already extremely accurate. Suitable candidates include few-body molecules such as H_2^+ and HD^+, which are among the benchmark systems discussed in the CODATA constants review as inputs and cross-checks across theory and precision measurement [17].

Concretely, one compares measured transition frequencies ν_{exp} with state-of-the-art theory predictions $\nu_{\mathrm{th}}(\alpha_{\mathrm{em}}, \mu, \ldots)$ (including QED, relativistic, and finite-size corrections as appropriate) and studies the residual

$$\Delta\nu := \nu_{\mathrm{exp}} - \nu_{\mathrm{th}}.$$

In the HPA interface viewpoint, any additional readout-consistency correction must enter through explicit, low-parameter interface quantities (e.g. a resolution proxy ε or an induced mismatch variable such as E_N in Appendix B), so that $\Delta\nu$ admits a structured decomposition rather than an unconstrained per-transition fit.

The falsifiability logic is:

- If experiments and standard theory agree within uncertainties, then either λ is effectively negligible at the relevant resolution, or the functional form of $\mathcal{D}_\varepsilon$ and S_{pf} must be constrained to yield suppressed corrections.
- If persistent residuals exist beyond known QED/nuclear corrections, then an interface model must explain not only magnitudes but also selection rules and systematic dependence on resolution-related proxies. Discrepancy-based corrections are highly structured and therefore restrictive.

In either case, the mismatch-cost proposal is testable.

8.3 What this paper does not claim

We do not claim that the closed-theory derivations alone replace quantum chemistry, nor that the interface functional (37) is already uniquely fixed. The claim is narrower: once $(\alpha_{\mathrm{em}}, \mu)$ are treated as protocol invariants, the two main structural prerequisites of chemistry (absolute scale and BO hierarchy) become necessary consequences of the same geometric data, and any additional readout-consistency physics must be encoded in explicit, falsifiable interface terms.

9 Conclusion

We reframed the "existence of chemistry" problem in the HPA–Ω scan–readout paradigm. In standard quantum chemistry, chemical bonding and molecular stability depend critically on two empirical dimensionless inputs: the fine-structure constant α_{em} and the proton–electron mass ratio μ. In the HPA layer discipline, these are instead treated as protocol-geometric invariants.

Using the companion constants-geometry results, we recorded the closed-theory theorem-level values $\alpha_{\mathrm{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi$ and $\mu_{\mathrm{geo}} = 6\pi^5$ and showed how they control the two structural pillars of chemistry: (i) α_{em} fixes atomic units and hence the absolute eV-scale of chemical energies, and (ii) μ fixes the BO small parameter $\mu^{-1/2}$ and thereby the hierarchical separation of electronic/vibrational/rotational spectra, molecular geometry stability, and isotope effects.

We further anchored these scale statements with rigorous stability-of-matter results, placing α_{geo} deep inside known stability regimes, and used the textbook H_2^+ Born–Oppenheimer curve as the minimal molecule-level reference that exhibits a bound minimum and the $\mu^{-1/2}$ control of nuclear zero-point motion. An optional mismatch-based interface template consistent with HPA phase thermodynamics is recorded in Appendix B; specifying and auditing concrete models for $\mathcal{D}_\varepsilon$ and S_{pf} is a natural next step.

A Audit table and reproducibility notes

A.1 Audit table: closed layer versus interface versus interpretation

Layer	Content used in this paper
Closed-theory	O5/O6 scan–readout interface (Axioms 2.1–2.2); discrepancy definitions; geometric impedance and phase-volume axioms (Sections 3–4); theorem-level values $\alpha_{\mathrm{geo}}^{-1} = 4\pi^3 + \pi^2 + \pi$ and $\mu_{\mathrm{geo}} = 6\pi^5$.
Interface	Matching inputs $s_\alpha = \exp(\Delta\alpha^{-1})$ and $s_\mu = \mu_{\mathrm{exp}}/\mu_{\mathrm{geo}}$, treated as explicit low-parameter interfaces to CODATA [5, 17].
Interpretation	Atomic-unit scaling, BO hierarchy, and the stability-window anchors used to interpret $(\alpha_{\mathrm{geo}}, \mu_{\mathrm{geo}})$ in chemistry language. An optional mismatch-based interface template is recorded separately in Appendix B.

A.2 Reproducible scripts

This paper directory includes a small helper script:

- `scripts/compute_constants.py`: computes $\alpha_{\mathrm{geo}}^{-1}$ and μ_{geo} and compares to CODATA central values (as recorded in [5]).

The scripts are written to be readable and auditable rather than maximally optimized.

B An optional interface template: phase stabilization under finite readout

This appendix records an *interface-level* model that connects the scan–readout viewpoint to chemistry language (bonding, reactions, and free-energy descent). It is not used in the closed-theory derivations of $(\alpha_{\mathrm{geo}}, \mu_{\mathrm{geo}})$, nor in the stability and scaling arguments of the main text; it is included as a constrained, falsifiable interface consistent with HPA phase thermodynamics.

B.1 A minimal mismatch-augmented functional

Let R denote nuclear coordinates (collectively). Let ψ be an electronic state normalized on the electronic Hilbert space at fixed R. Define the functional

$$\mathcal{F}[\psi;R] := \langle\psi|\hat{H}_{\mathrm{Coul}}(\alpha_{\mathrm{em}};R)|\psi\rangle \;+\; \lambda\,\mathcal{D}_\varepsilon[\psi;R] \;-\; T_c\,S_{\mathrm{pf}}[\psi;\varepsilon], \tag{37}$$

where $\hat{H}_{\mathrm{Coul}}$ is the standard Coulomb electronic Hamiltonian, $\mathcal{D}_\varepsilon$ is a resolution-dependent mismatch penalty, and S_{pf} is a phase-friction entropy proxy (e.g. discrepancy-based) [11]. For fixed R, define the corresponding effective surface

$$E_{\mathrm{eff}}(R) = \inf_{\psi} \mathcal{F}[\psi;R]. \tag{38}$$

In the formal limit $\lambda \to 0$ and when the entropy term is negligible, E_{eff} reduces to the standard BO surface used in quantum chemistry.

B.2 Bonding and reaction direction (interpretation)

In standard chemistry language, a bond forms when the electronic energy is lowered by delocalization and exchange at an intermediate nuclear separation, producing a minimum in $E_{\mathrm{BO}}(R)$. In the HPA interface language, the same phenomenon can be phrased as:

- **Bond formation**: reconfiguration that lowers the total protocol cost by stabilizing phases (reducing mismatch accumulation) under finite readout.
- **Reaction direction**: evolution toward configurations that decrease the effective functional (37), aligning with a free-energy descent structure but expressed via phase-friction entropy and mismatch penalties [11,27].

The purpose of (37) is falsifiability: once a concrete $\mathcal{D}_\varepsilon$ and S_{pf} are specified, the implied corrections to molecular levels must exhibit structured parameter dependence tied to readout resolution and discrepancy, rather than arbitrary higher-order operators.

B.3 A minimal computable toy choice (for falsifiability bookkeeping)

To make (37) operational without committing to a full microscopic derivation, we record a minimal interface that is explicit, low-parameter, and auditable:

- Choose a finite readout depth N associated with resolution ε (e.g. $N \sim \varepsilon^{-1}$ as a bookkeeping proxy).
- Let the scan orbit be $x_n = x_0 + n\nu \pmod 1$ from Axiom 2.2.
- Define a phase-friction proxy by $S_{\mathrm{pf}}(N) = k_B\,E_N$ with $E_N = ND_N^*$ as in Section 2.
- Define a mismatch penalty as a scalar multiple of the same mismatch accumulation,

  $$\mathcal{D}_\varepsilon := E_N,$$

  so that the interface correction is controlled by a single effective coefficient $(\lambda - T_c k_B)$ at fixed (ε, N).

This minimal choice fixes a concrete residual template so that any spectroscopy-facing comparison can be phrased in terms of an explicit mismatch variable E_N rather than an unspecified functional; more detailed interfaces should reduce to an equivalent low-parameter form at leading order.

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