Biological Computational Teleology in HPA–Ω: Life as Predictive Active Error Correction Against Phase Friction

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Abstract

Life is often described as an emergent far-from-equilibrium structure sustained by chemical dissipation. In the HPA–Ω program, we adopt a strict layer discipline: the ontic layer is a global unitary scan, while irreversibility and thermodynamic entropy arise operationally from finite-resolution scan–projection readout [1–4]. This paper proposes a computable and auditable definition of life in that interface language: **life is a predictive active error-correction (AEC) system**. Its core function is to use information acquisition, internal modeling, and feedback control to reduce its own *phase-friction* entropy production (relative to a passive baseline) by exporting dissipation to external waste channels.

In arithmetic statistical mechanics (ASM) [4], phase friction over a length-N window is certified by star discrepancy D_N^* of the induced phase-point set, with accumulated mismatch $E_N := ND_N^*$ and the phase-friction entropy certificate $S_{\mathrm{pf}}(N) := k_{\mathrm{B}}E_N$. A geometric Landauer principle [5,6] together with information-thermodynamic bounds [7,8] yields necessary survival inequalities: $\dot{F}_{\mathrm{pred}} \le k_{\mathrm{B}}T_c\,\dot{I}_{\mathrm{pred}}$ and $\dot{F}_{\mathrm{pred}} > \dot{W}_{\mathrm{diss}}$, hence $\dot{I}_{\mathrm{pred}} > \dot{W}_{\mathrm{diss}}/(k_{\mathrm{B}}T_c)$. We define predictive efficiency $\eta_{\mathrm{pred}} := \dot{F}_{\mathrm{pred}}/\dot{W}_{\mathrm{diss}}$ and recast genetic coding, homeostasis, and evolution as multi-scale strategies that optimize η_{pred} under readout and architectural constraints. Finally, we propose a falsifiable control-law hypothesis for biological rhythms: to resist low-order phase locking, adaptive coupling ratios should be biased toward badly-approximable irrational numbers, with the golden branch φ^{-1} as the Hurwitz extremal candidate [9–11].

Keywords: HPA–Ω; arithmetic statistical mechanics; phase friction; active error correction; reverse compilation; predictive information rate; geometric Landauer principle; coupled oscillators; golden ratio; computational teleology.

Layer discipline and conventions. Unless otherwise stated, log denotes the natural logarithm and "mod 1" refers to reduction in $\mathbb{R}/\mathbb{Z} \cong \mathbb{T}$. "Ontic" refers to the global unitary-scan layer, while "operational" refers to finite-resolution readout and implementation constraints. We use N for finite-window length and reserve $\mathcal{N}$ for computational lapse.

Layered audit rule. We separate a *mathematical layer* (definitions and computable protocol-level statements, such as discrepancy-based certificates and information-thermodynamic bounds) from a *biological identification layer* (interface mappings to concrete biochemical or physiological mechanisms, stated in a falsifiable form). Appendix G records a shared interface template that connects stable-sector descriptions (stability selection under protocol constraints) with AEC descriptions (active suppression of mismatch relative to a passive baseline).

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Contents

1 Introduction: from “negative entropy” to phase friction, from fitness to predictive efficiency

Schrödinger famously characterized life as “feeding on negative entropy” [12]. The statement is operationally correct—living systems are open and must dissipate—but it does not by itself answer a sharper question: *what is life, as a physical mechanism, in a theory where the ontic microdynamics is unitary and therefore reversible?* Standard thermodynamics can quantify entropy production, yet it treats irreversibility at the operational level via coarse graining and macroscopic state variables [13]. The HPA–Ω program instead locates irreversibility at a specific interface: finite-resolution scan–projection readout [1–3]. In this view, “temperature,” “probability,” and entropy increase are not ontic primitives but operational consequences of the readout channel.

From readout mismatch to phase friction. Arithmetic statistical mechanics (ASM) makes this interface explicit by identifying a computable mismatch certificate between a continuous phase orbit and discrete readout statistics [4]. For a Weyl scan with slope α, the readout induces a finite point set in $[0, 1)$; the star discrepancy D_N^* quantifies the worst-case deviation from uniformity over a length-N window, and the accumulated mismatch $E_N := ND_N^*$ defines the operational phase-friction entropy certificate $S_{\mathrm{pf}}(N) := k_{\mathrm{B}}E_N$. The key structural bifurcation is arithmetic: rational slopes lock into finite periodic orbits with linear mismatch growth, while badly-approximable irrational slopes admit logarithmic mismatch growth and are maximally anti-locking in the Hurwitz/Markov sense [9, 14].

Stable sectors and correction as two faces of the same interface. Finite-resolution readout induces symbolic words and therefore admits a complementary viewpoint in which explicit protocol constraints select a stable visible sector (a compressed type set) from a larger microstate alphabet. The present manuscript emphasizes the control-theoretic dual: rather than assuming stability as a primitive, an agent maintains stability by predictive modeling and feedback that reduce mismatch relative to a passive baseline. Appendix G records a protocol-level dictionary connecting these two languages (stability selection and active correction) within the same layered audit rule [15, 16].

Life as predictive active error correction (AEC). Once irreversibility is tied to a concrete mismatch mechanism, a minimal engineering fact becomes unavoidable: long-lived low-entropy structure requires *continuous repair*. This paper proposes a strict operational definition: *life is a predictive AEC subsystem that reduces its own phase-friction entropy production rate relative to a passive baseline by acquiring information, maintaining an internal model, and applying feedback, while paying the required dissipation cost.* This definition does not invoke metaphysical teleology; rather, it derives an *engineering teleology* from readout constraints and information thermodynamics: a living system must keep a positive margin between predictive free-energy gain and dissipation required for repair.

Main contributions.

- We formulate an auditable interface definition of life/agency in HPA–Ω as predictive active error correction against readout-induced phase friction (Section 4).
- We derive necessary survival inequalities from a geometric Landauer principle and information-thermodynamic bounds, and introduce predictive efficiency as an evolutionary performance metric (Section 5).

- We propose a falsifiable control-law hypothesis for biological rhythms: adaptive coupling ratios should be biased toward badly-approximable irrationals, with the golden branch as the extremal anti-locking candidate (Section 7).
- We give concrete experimental/data-analysis interfaces (Section 11) and provide minimal reference implementations for computing the core certificates and statistics (Appendix H).

Scope and posture. This manuscript does not claim to derive terrestrial biochemistry from first principles. Instead, it provides a *layered interface language* that constrains what any sustainable "life-like" subsystem must do in a scan–projection universe: it must implement predictive control at a sufficient information rate and at a sufficient dissipation budget. Biological chemistry is then a particular high-performance implementation of that control problem.

2 Readout-induced entropy and the necessity of active repair

2.1 Scan–readout two-layer structure and Weyl complementarity

The HPA–Ω axioms separate an ontic unitary scan from an operational readout interface [1–3]. In the minimal model, the scan and the pointer phase form a Weyl pair (U_{scan}, V) with commutation relation

$$U_{\mathrm{scan}}V = \mathrm{e}^{2\pi\mathrm{i}\alpha}\, V U_{\mathrm{scan}}, \qquad \alpha \in (0,1)\setminus\mathbb{Q}. \tag{1}$$

In the standard representation on $L^2(\mathbb{T})$, the scan induces an irrational rotation orbit [17]

$$x_n = x_0 + n\alpha \pmod 1, \qquad n \in \mathbb{N}, \tag{2}$$

while the readout extracts only finite-resolution statistics from that orbit. Because U_{scan} and V do not commute, finite-resolution readout cannot access "time" (scan index) and "phase" arbitrarily sharply at the same time; operational irreversibility is therefore an interface phenomenon rather than an ontic one.

Finite-resolution readout. Operationally, we model readout as a finite-resolution instrument or POVM $\{E_k^{(\varepsilon)}\}$ indexed by a resolution parameter $\varepsilon > 0$, with $\sum_k E_k^{(\varepsilon)} = \mathbb{1}$ [18]. Each outcome corresponds to a coarse window $w_k^{(\varepsilon)}$ over the pointer phase, inducing a CPTP map from the ontic state to an effective classical distribution. The exact choice of kernel is part of the experimental protocol; the key point is structural: *finite resolution injects a systematic mismatch between the continuous orbit and discrete distinguishable outcomes.*

2.2 Arithmetic statistical mechanics: discrepancy and accumulated mismatch

Given the induced phase points $\{x_1, \ldots, x_N\} \subset [0,1)$, define the empirical distribution function

$$F_N(a) := \frac{1}{N}\#\{1 \le n \le N :\ x_n < a\}, \qquad 0 \le a \le 1, \tag{3}$$

and the *star discrepancy*

$$D_N^* := \sup_{0\le a\le 1} \left|F_N(a) - a\right|. \tag{4}$$

The discrepancy measures the worst-case deviation between the readout-induced finite sample and the ideal uniform reference on $[0,1)$. ASM uses the *accumulated mismatch*

$$E_N := N D_N^* \tag{5}$$

as a protocol-auditable certificate of readout inconsistency accumulation [4,14,19]. The phase-friction entropy certificate on a length-N window is defined as

$$S_{\mathrm{pf}}(N) := k_{\mathrm{B}} E_N. \tag{6}$$

This is not a claim that all thermodynamic entropy *reduces* to one-dimensional discrepancy; rather, it provides a minimal computable certificate that is robust under protocol composition and captures the key arithmetic bifurcation (rational locking vs irrational spreading).

Why star discrepancy is the natural certificate. Star discrepancy controls worst-case readout deviations for all threshold-type coarse measurements, and it also controls integration error for bounded-variation observables via the Koksma–Hlawka inequality [14, 19]: for any $f : [0, 1) \to \mathbb{R}$ of bounded variation $V(f)$,

$$\left| \frac{1}{N} \sum_{n=1}^{N} f(x_n) - \int_0^1 f(x)\,\mathrm{d}x \right| \le V(f)\, D_N^*. \tag{7}$$

Thus D_N^* is an auditable, protocol-level upper bound on worst-case readout bias across a large class of coarse observables. Moreover, in one dimension the discrepancy over *all* intervals is controlled by star discrepancy up to a universal factor (e.g. $D_N \le 2D_N^*$) [14], so D_N^* is a sufficient certificate at the level of order-of-magnitude entropy production.

2.3 A "third-law template": zero friction is unattainable without locking

Discrepancy theory implies a stark dichotomy for Kronecker orbits (2) [9,14]. If α is rational, the orbit is periodic, and mismatch accumulates linearly. If α is badly approximable (equivalently: its continued-fraction partial quotients are bounded), discrepancy is controlled and E_N grows only logarithmically.

Proposition 2.1 (Rational locking yields linear mismatch growth). *If $\alpha = p/q \in \mathbb{Q}$ in lowest terms, then the orbit* (2) *visits only q distinct points and one has the universal lower bound*

$$D_N^* \ge \frac{1}{2q}, \qquad E_N \ge \frac{N}{2q} \tag{8}$$

for infinitely many N (in particular for multiples of q) [14, 19].

Theorem 2.2 (Continued-fraction certificate for star discrepancy). *Let $\alpha \in (0, 1) \setminus \mathbb{Q}$ have continued fraction expansion $\alpha = [0; a_1, a_2, \ldots]$ with convergent denominators $(q_m)_{m \ge 0}$. For any $N \in \mathbb{N}$, choose m such that $q_m \le N < q_{m+1}$. Then for the Kronecker sequence* (2) *one has the explicit bound [14, 19]*

$$D_N^* \le \frac{1 + \sum_{i=1}^{m} a_i}{N}, \qquad \textit{equivalently} \qquad E_N \le 1 + \sum_{i=1}^{m} a_i. \tag{9}$$

Corollary 2.3 (Explicit logarithmic mismatch bound for bounded partial quotients). *If α has bounded continued-fraction partial quotients, i.e. $a_i \le A$ for all i, then for all $N \ge 1$,*

$$E_N \le 1 + A\, m(N) \le 1 + A \left\lceil \log_\varphi\!\left(\sqrt{5}\, N\right) \right\rceil, \tag{10}$$

where $m(N)$ is the unique index such that $q_{m(N)} \le N < q_{m(N)+1}$ and $\varphi = (1 + \sqrt{5})/2$. In particular, $E_N = O(\log N)$ with an explicit constant.

Corollary 2.4 (Golden-branch bound (fully explicit)). *For the golden branch $\alpha = \varphi^{-1} = [0; 1, 1, 1, \ldots]$, the convergent denominators are Fibonacci numbers $q_m = F_{m+1}$, hence for all $N \ge 1$,*

$$E_N \le 1 + \left\lceil \log_\varphi\!\left(\sqrt{5}\, N\right) \right\rceil. \tag{11}$$

In this sense, "zero phase friction" is not a stable operational limit: pushing the system toward exact periodicity (rational locking) collapses openness into a finite cycle and creates a linear mismatch channel. For long-term sustainability, a system must instead operate in the badly-approximable regime where mismatch is unavoidable but controllable. The golden branch $\alpha = \varphi^{-1}$ is extremal in the Hurwitz/Markov sense and provides a canonical anti-locking reference [9].

3 From mismatch certificates to entropy production rates and computational thermodynamics

Section 2 defined phase friction over a finite window via discrepancy and accumulated mismatch. To connect that certificate to biological maintenance costs, we need rate-level quantities.

3.1 Mismatch density and phase-friction entropy production

Let τ denote intrinsic scan time (iteration count). In a spatially extended setting, let x denote a location or subsystem label. Fix a readout phase coordinate $u \in [0, 1)$ (constructed from the experimental readout protocol) and a window length N. At external time t, let $\{u_{t,1}, \dots, u_{t,N}\}$ be the N phase points produced by the protocol over a window ending at t, and define the windowed certificates $D_N^*(t)$, $E_N(t) := N D_N^*(t)$, and

$$S_{\mathrm{pf}}^{(N)}(t) := k_{\mathrm{B}} E_N(t). \tag{12}$$

The phase-friction entropy flow is then defined by a finite-difference rate:

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}t}(t) := \lim_{\Delta t \to 0} \lim_{N \to \infty} \frac{S_{\mathrm{pf}}^{(N)}(t + \Delta t) - S_{\mathrm{pf}}^{(N)}(t)}{\Delta t}, \tag{13}$$

with N and Δt chosen by the protocol and then smoothed at the analysis layer. This expression should be read as an analysis-layer idealization, not as a requirement to take literal limits in empirical work. Operationally, one fixes a readout uniformization map $y \mapsto u \in [0, 1)$, a window length N, a step size Δt, and a smoothing operator, and then works with finite estimators extracted from $E_N(t)$. All cross-system comparisons are meaningful only under matched protocol choices (same uniformization, N, Δt, and smoothing). Section 11 records concrete data-analysis pipelines and decision criteria built from these finite estimators. In intrinsic scan time, we define the *mismatch density* (phase-friction entropy production density) as

$$\sigma(x, \tau) := \frac{1}{k_{\mathrm{B}}} \frac{\mathrm{d}S_{\mathrm{pf}}(x; \tau)}{\mathrm{d}\tau}, \tag{14}$$

so that

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}\tau} = k_{\mathrm{B}}\, \sigma(x, \tau). \tag{15}$$

Operationally, σ is a rate extracted from the windowed mismatch certificate (12) via (13); it is not a phenomenological noise parameter.

3.2 Computational lapse and externally observed entropy flow

HPA–Ω introduces a *computational lapse* $\mathcal{N}(x)$ determined by local routing overhead $\kappa(x)$ in the implementation dictionary [2, 20, 21]. Relative to a reference overhead κ_0, define

$$\mathcal{N}(x) := \frac{\kappa_0}{\kappa(x)}. \tag{16}$$

Operationally, $\mathcal{N}$ is the protocol-level rescaling between external time t and local scan time τ:

$$\mathrm{d}\tau_{\mathrm{loc}}(x) = \mathcal{N}(x)\,\mathrm{d}t. \tag{17}$$

Combining (15) with (17) yields the externally observed entropy-flow law

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}t} = \frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}\tau_{\mathrm{loc}}}\frac{\mathrm{d}\tau_{\mathrm{loc}}}{\mathrm{d}t} = k_{\mathrm{B}}\,\sigma(x,\tau)\,\mathcal{N}(x). \tag{18}$$

It is convenient to introduce the external-time mismatch rate

$$\Sigma(x,t) := \sigma(x,\tau)\,\mathcal{N}(x), \tag{19}$$

so that $\mathrm{d}S_{\mathrm{pf}}/\mathrm{d}t = k_{\mathrm{B}}\,\Sigma(x,t)$. For biological systems, Eq. (18) supplies a unifying interface: different tissues and environments can exhibit different effective computational lapse profiles (estimated from routing/transport/control overhead proxies), which rescale the entropy flow observed per unit external time.

3.3 Computational temperature and the cost of control

In information thermodynamics, temperature sets the energetic scale of information processing costs and feedback advantages [5, 7]. HPA–Ω introduces an effective *computational temperature* T_c associated to the readout/implementation interface [4]. Operationally, T_c is the coefficient that converts erased information into a minimal work/heat scale at the readout interface (Landauer scale): erasing ΔI nats costs at least $k_{\mathrm{B}}T_c\,\Delta I$ (up to additional geometric impedance), and erasing one bit costs at least $k_{\mathrm{B}}T_c \ln 2$. This T_c need not equal a physical bath temperature; rather, it is the operational temperature relevant for control and erasure at the interface. In the next sections we use T_c to formulate necessary inequalities for predictive control to offset phase-friction dissipation.

4 Operational definition: life as predictive active error correction

Phase friction is unavoidable at finite resolution (Section 2). Therefore, any subsystem that maintains stable low-entropy structure over long times must implement continuous repair. This motivates an operational definition of life that is compatible with the second law and is auditable at the protocol level.

4.1 Agent definition at the readout interface

Definition 4.1 (Agent as predictive active error correction (AEC))**.** *Fix a finite-resolution readout protocol and an external environment. Let S be an open subsystem with a bounded computational flux budget $\mathcal{E}$, understood operationally as an upper bound on the sustained average control power available for measurement, modeling, and feedback. We call S an* agent *if there exist internal state variables M (a memory/model) and feedback operations $\mathcal{U}$ such that:*

1. *(**conditional information acquisition**) the readout channel provides conditional information about the environment, yielding a nontrivial mutual information between M and environment-dependent readout outcomes;*

2. *(**predictive internal model**) the internal dynamics maintains a predictive model of future readout outcomes (possibly coarse-grained), so that M carries information about future readout statistics;*

3. *(**active reduction of phase friction**) applying $\mathcal{U}$ reduces the subsystem's phase-friction entropy production rate relative to a passive baseline (same protocol and environment, but with feedback disabled), i.e.*

$$\dot{S}_{\rm pf}^{(\rm active)} \; < \; \dot{S}_{\rm pf}^{(\rm passive)} \tag{20}$$

on a sustained time interval, while respecting the budget $\mathcal{E}$.

Remark 4.2 (Life as an AEC phase)**.** *Definition 4.1 is intentionally minimal and interface-level: it does not identify life with a particular chemistry. In this manuscript we use "life" as shorthand for an AEC-capable agent that operates stably over long time scales, i.e. one that can maintain low-entropy structure despite persistent phase friction. The definition is compatible with standard thermodynamics: an agent does not destroy entropy; it* redirects *dissipation into waste channels while protecting internal degrees of freedom. In practice, the inequality in Definition 4.1 is tested using the mismatch-rate estimator $\Sigma_N(t)$ derived from windowed discrepancy certificates (Section 3.1).*

4.2 Reverse compilation: prediction as arithmetic compression, control as phase correction

In the Ω implementation dictionary, local unitary update rules can be compiled into nearest-neighbor circuits, and the required depth defines routing overhead $\kappa(x)$ and lapse $\mathcal{N}(x)$ (Section 3.2). We use "forward compilation" for the map from dynamics to implementation cost. The agent operation is the reverse direction: given limited internal bits, infer what future readout will produce and pre-configure corrective actions.

Reverse compilation (interface description). At the operational layer, prediction is the compression of future readout words under a finite alphabet and finite resolution. Control is the implementation of phase corrections compatible with the Weyl structure (1). Together they aim to reduce mismatch density $\sigma(x,\tau)$ and therefore reduce the phase-friction entropy flow (18).

Canonical coding in the golden branch. For irrational rotations, canonical codings (Ostrowski, Zeckendorf/Fibonacci in the golden branch) provide a natural coordinate system for readout compression and for multi-scale stabilization [1, 4, 21, 22]. In this sense, "life is not a collection of chemical reactions": chemistry is a physical substrate, while life is the algorithmic phase of reverse compilation—predicting future readout and actively correcting phase-friction mismatch.

5 Thermodynamic bounds: geometric Landauer, predictive gain, and a survival inequality

An AEC agent must pay for error correction; otherwise Definition 4.1 would contradict the second law. This section summarizes the relevant lower bounds and yields a necessary survival inequality in terms of predictive information rate.

5.1 Geometric Landauer principle: correction costs depend on architecture

Landauer's principle bounds the minimal heat generated by logically irreversible operations such as erasure [5, 6]. In HPA–Ω, the relevant operational temperature is the computational temperature T_c (Section 3.3). Moreover, erasure and re-encoding occur on a constrained physical network (locality, transport, routing), so there is an additional architecture-dependent cost [4]:

$$W_{\rm erase} \;\geq\; k_{\rm B} T_c \ln 2 \;+\; Z_{\rm geom}. \tag{21}$$

Here Z_{geom} is a geometric impedance term that accounts for routing overhead, localization constraints, and reconfiguration costs. For biology, Eq. (21) means that the energetic price of repair depends not only on bit counts but also on physical organization: transport distances, congestion, connectivity topology, and control latency all affect the minimal dissipation required for maintaining reliable structure.

5.2 Predictive gain upper bound from information thermodynamics

Measurement–feedback thermodynamics bounds the maximum work/free-energy advantage obtainable from information [7, 8]. At the operational level, we write the predictive free-energy gain rate as $\dot{F}_{\text{pred}}$ and the predictive mutual information rate between internal state M and future readout outcomes as $\dot{I}_{\text{pred}}$. Then a generic bound of the form

$$\dot{F}_{\text{pred}} \;\le\; k_{\text{B}} T_c \, \dot{I}_{\text{pred}} \tag{22}$$

holds when $\dot{I}_{\text{pred}}$ is measured in nats per unit time (our convention $\log = \ln$). If $\dot{I}_{\text{pred}}$ is measured in bits per unit time, the corresponding bound is

$$\dot{F}_{\text{pred}} \;\le\; k_{\text{B}} T_c \ln 2 \, \dot{I}_{\text{pred}}^{(\text{bits})}. \tag{23}$$

Eq. (22) makes "prediction" an accountable resource: without sufficient predictive information rate, no agent can extract a sustained free-energy advantage to fund repair.

5.3 A necessary survival inequality and predictive efficiency

Let $\dot{W}_{\text{diss}}$ denote the dissipation rate required to counter phase friction and maintain structure (including the geometric impedance costs implicit in Z_{geom}). A necessary condition for sustained existence of the AEC phase is that predictive gain exceeds dissipation:

$$\dot{F}_{\text{pred}} \;>\; \dot{W}_{\text{diss}}. \tag{24}$$

Minimal dissipation imposed by phase-friction entropy flow. Independent of any particular biological mechanism, if phase-friction entropy is produced at rate $\mathrm{d}S_{\text{pf}}/\mathrm{d}t$ and is exported through a channel characterized by computational temperature T_c, then the second law requires a minimal heat/work outflow of order $T_c \, \mathrm{d}S_{\text{pf}}/\mathrm{d}t$. Using Eq. (18) and the definition (19), this yields the protocol-level lower bound

$$\dot{W}_{\text{diss}} \;\ge\; T_c \frac{\mathrm{d}S_{\text{pf}}}{\mathrm{d}t} \;=\; k_{\text{B}} T_c \, \Sigma(x,t). \tag{25}$$

We interpret this as a Clausius-type bound at the readout interface [13, 23].

Information-rate threshold (explicit form). Combining (22), (24), and (25) yields the predictive-information-rate threshold

$$\dot{I}_{\text{pred}} \;>\; \frac{\dot{W}_{\text{diss}}}{k_{\text{B}} T_c} \;\ge\; \Sigma(x,t). \tag{26}$$

We define the *predictive efficiency*

$$\eta_{\text{pred}} := \frac{\dot{F}_{\text{pred}}}{\dot{W}_{\text{diss}}}. \tag{27}$$

In this interface language, biological "purpose" becomes a computable engineering objective: maximize η_{pred} under resource constraints so that $\eta_{\text{pred}} > 1$ is sustainable over the relevant time scales.

6 Multi-scale realizations of AEC: from cells to organisms

The AEC definition (Section 4) is an interface statement. This section maps it to familiar biological mechanisms as concrete realizations of information acquisition, predictive modeling, and feedback correction.

6.1 Cellular scale: homeostasis as error-correction budget management

At the cellular scale, the three elements of Definition 4.1 have direct correspondences:

- **Information acquisition.** Receptor signaling, metabolic sensing, and damage sensing are finite-resolution readouts with thresholds, noise, and discretization. They produce conditional information about external conditions and internal state [24].
- **Internal model.** Gene-regulatory networks and epigenetic states encode compressed priors mapping environments to phenotypes; operationally, they approximate future readout distributions under finite resources [24].
- **Feedback correction.** Proteostasis (folding quality control, chaperones, ubiquitin–proteasome degradation), DNA replication proofreading and repair, membrane-potential maintenance, and stress responses are all mechanisms that reduce effective mismatch density $\sigma(x, \tau)$ by actively exporting dissipation to waste heat and waste material [24–27].

The HPA–Ω framing emphasizes that these processes are not optional "luxuries"; they are forced by the inevitability of phase friction at finite resolution.

6.2 Organism scale: behavior and nervous systems as higher-level reverse compilers

At the organism scale, reverse compilation (Section 4.2) appears as predictive perception and action:

- **Sensorimotor loops.** Sensory streams provide conditional information, internal circuits encode predictive models of future sensory outcomes, and actions implement feedback that shapes future readout [28, 29].
- **Learning and memory.** Learning increases predictive mutual information rate $\dot{I}_{\text{pred}}$ by compressing regularities, while memory maintenance and updating incur geometric Landauer costs that depend on circuit architecture (Eq. (21)) [29].
- **Behavioral teleology.** Strategy selection can be reinterpreted as maximizing long-run predictive efficiency η_{pred} (Eq. (27)) under metabolic and architectural constraints.

This does not replace evolutionary fitness; it provides a physical decomposition of fitness into an information-rate budget and a dissipation budget, yielding a measurable interface between biology and readout thermodynamics.

7 Golden-branch stabilizer: an anti-locking control-law hypothesis for biological rhythms

If phase friction is tied to arithmetic mismatch, then a dominant failure mode of adaptive oscillatory control is low-order phase locking: rational resonances generate short pseudo-periods and amplify mismatch accumulation (Proposition 2.1). Coupled-oscillator theory organizes locking near rationals by Arnold tongues, with low-denominator resonances typically having the widest tongues and therefore being easiest to lock into at fixed noise/coupling strength [10, 11].

7.1 Number-theoretic reason: the golden branch is maximally badly approximable

For any irrational α, Diophantine approximation studies the quality of rational approximations p/q. Badly-approximable numbers are those with a uniform lower bound of order $1/q^2$. The golden branch $\alpha = \varphi^{-1} = (\sqrt{5}-1)/2$ is extremal: among irrationals it maximizes the uniform constant in the $1/q^2$ bound (Hurwitz/Markov extremality) [9]. This makes it the canonical anti-locking ratio under finite tolerance.

7.2 A quantitative anti-locking index

Fix an operational locking tolerance $\delta > 0$ that summarizes noise level, coupling strength, or measurement resolution. Define the resonance susceptibility index

$$Q_\delta(\alpha) := \min\left\{ q \in \mathbb{N} : \ \exists p \in \mathbb{Z} \text{ s.t. } \left|\alpha - \frac{p}{q}\right| < \delta \right\}. \tag{28}$$

If $Q_\delta(\alpha)$ is large, the system must reach high-order resonances (large denominator) before it can lock within tolerance δ.

Proposition 7.1 (Diophantine lower bound for the anti-locking index)**.** *If α is badly approximable, then there exists $c(\alpha) > 0$ such that*

$$\left|\alpha - \frac{p}{q}\right| \geq \frac{c(\alpha)}{q^2} \qquad \text{for all } \frac{p}{q} \in \mathbb{Q} \tag{29}$$

[9]. For any tolerance $\delta > 0$, this implies

$$Q_\delta(\alpha) \ \geq \ \left\lceil \left(\frac{c(\alpha)}{\delta}\right)^{1/2} \right\rceil. \tag{30}$$

In particular, for $\alpha = \varphi^{-1}$ one can take $c(\varphi^{-1}) = 1/\sqrt{5}$, yielding the sharp bound

$$Q_\delta(\varphi^{-1}) \ \geq \ \left\lceil \left(\frac{1}{\sqrt{5}\,\delta}\right)^{1/2} \right\rceil. \tag{31}$$

7.3 Operational identification of the tolerance δ

The tolerance parameter δ is not a metaphysical knob; it is an operational summary of how wide low-order resonances are under noise and coupling. In standard phase-reduction theory, two weakly coupled oscillators admit an Adler-type phase-difference equation [10,11]. For a near $p{:}q$ resonance, define the resonant phase $\psi := p\theta_1 - q\theta_2$; then the reduced dynamics has the form

$$\dot{\psi} = \Delta_{p:q} + K_{p:q}\, g(\psi) + \text{noise}, \tag{32}$$

where $\Delta_{p:q} := p\omega_1 - q\omega_2$ is the detuning and $K_{p:q}$ is an effective coupling amplitude. In the noise-free case, locking occurs when $|\Delta_{p:q}|$ is below an order-$K_{p:q}$ threshold. Converting detuning into ratio space, with $\alpha := \omega_1/\omega_2$ one has

$$\left|\alpha - \frac{p}{q}\right| = \frac{|\Delta_{p:q}|}{q\,\omega_2} \ \lesssim \ \frac{K_{p:q}}{q\,\omega_2}, \tag{33}$$

which provides an operational route to estimate δ (or a q-dependent tolerance $\delta_{p:q}$) from inferred coupling strengths and noise levels.

7.4 Null baseline: typical scaling of Q_δ for random ratios

To turn Prediction P1 into a quantitative statistical test, one needs a baseline for Q_δ in the absence of selection pressure. If α is drawn uniformly from $(0,1)$, then $Q_\delta(\alpha) \le Q$ iff α lies within δ of some reduced rational p/q with $1 \le q \le Q$. A simple union bound yields

$$\mathbb{P}(Q_\delta(\alpha) \le Q) \;\le\; 2\delta \sum_{q=1}^{Q} \phi(q), \tag{34}$$

where $\phi(q)$ is Euler's totient function counting reduced residues. Using the classical summatory estimate $\sum_{q\le Q} \phi(q) = \frac{3}{\pi^2}Q^2 + O(Q\log Q)$ [30], one obtains the asymptotic baseline

$$\mathbb{P}(Q_\delta(\alpha) \le Q) \;\lesssim\; \frac{6}{\pi^2}\,\delta\, Q^2 \qquad (\delta Q^2 \ll 1), \tag{35}$$

so the typical scale of Q_δ under the null is $Q_\delta \asymp \delta^{-1/2}$. This suggests a scale-free normalization for cross-system comparison:

$$Z_\delta(\alpha) := \sqrt{\delta}\, Q_\delta(\alpha). \tag{36}$$

At the leading-order null scale, the threshold $\frac{6}{\pi^2}\delta Q^2 \sim 1$ corresponds to $Z_\delta \sim \pi/\sqrt{6}$. Prediction P1 therefore becomes a concrete tail test: at matched δ, biological data should show systematically larger Q_δ than the $\delta^{-1/2}$ null scaling.

7.5 Prediction P1: statistical bias of rhythm ratios toward anti-locking irrationals

We can now state a falsifiable control-law hypothesis.

Remark 7.2 (Control-law hypothesis (anti-locking selection pressure))**.** *Consider an adaptive biological oscillator network whose primary failure mode is low-order phase locking (synchrony collapse, limit-cycle trapping, energy blow-up due to repeated mismatch accumulation). If selection pressure favors operating points that remain unlockable within the effective tolerance δ, then inferred effective coupling ratios α should be biased toward badly-approximable irrationals and should exhibit systematically larger $Q_\delta(\alpha)$ than appropriate randomized baselines. The golden branch is the Hurwitz extremal candidate and therefore a natural reference point.*

This hypothesis is not a claim that "the golden ratio is everywhere"; it is a task- and tolerance-dependent statistical prediction. Section 11 gives a concrete data-analysis pipeline and a reproducible computation of Q_δ (Appendix H).

8 Genetic information as readout arithmetic: discretization, coarse graining, and correction

HPA–Ω is basis-independent at the interface level: "scan–projection readout" is a structural constraint, not a particular physical substrate. This motivates a biological interface hypothesis: persistent biological information systems may exhibit architectures that look like finite-resolution readout plus active correction.

8.1 Interface hypothesis: genetic systems as finite-resolution readout–correction channels

Remark 8.1 (Interface hypothesis for genetic coding)**.** *The genetic alphabet and its decoding pipeline are modeled as a finite-resolution readout map from a large microscopic configuration space (molecular conformations, binding fluctuations, chemical noise) to a discrete executable* *output space (amino-acid sequences and regulatory actions). Degeneracy of the code (many codons mapping to one amino acid) is a coarse-graining that increases robustness under read-out noise, while proofreading and repair mechanisms provide active correction to maintain low effective mismatch [24–27, 31].*

We treat this as an interface-level mapping whose scientific content is carried by operational predictions and measurable proxies. It does, however, align with three basic observations:

- **Finite alphabet with combinatorial capacity.** A small discrete alphabet supports large effective state spaces through composition (e.g. triplet codons).
- **Degeneracy as controlled coarse graining.** Many-to-one mappings (including wobble pairing) trade fine detail for robustness [31].
- **Correction is not free.** Proofreading and repair require energy and physical transport; therefore their minimal costs must include a Landauer scale and an architecture-dependent geometric impedance (Eq. (21)).

8.2 Genetic memory as compressed predictive prior

In the AEC language, genetic information is not a static "blueprint" but a compressed prior that supports prediction and control across time: it constrains the space of internal models M that an organism can instantiate and maintain under finite resources. Maintenance of that prior (replication fidelity, error correction, epigenetic stabilization) consumes dissipation budget $\dot{W}_{\rm diss}$ and therefore competes with other uses of energy such as growth and reproduction. This makes genetics part of the same optimization problem as behavior: allocate limited dissipation and limited predictive information capacity to sustain $\eta_{\rm pred}$ in the environment.

9 Evolution as computational teleology: selection as predictive-efficiency optimization

In the HPA–Ω program, "teleology" is not a metaphysical goal but an interface-level resource statement: under finite readout and finite implementation budgets, stable histories must balance expressive openness against readability/auditability [21]. For living systems, this balance appears locally as a requirement to avoid both trivial periodic locking (loss of openness) and unstructured noise (loss of readability/predictability).

9.1 A protocol-stable selection template

Protocol-stable period data and computational teleology propose a generic selection template: in a finite time horizon N, define an auditable cost $J_N(\theta)$ capturing accumulated mismatch (or an explicitly defined estimator derived from it) and a complexity/implementation cost $C_{\rm comp}(\theta)$ capturing geometric impedance and maintenance burdens [32]. A minimal teleological dynamics takes the form of a dissipative gradient flow

$$\dot{\theta} \;=\; -\nabla_\theta\Big(J_N(\theta) + \beta\, C_{\rm comp}(\theta)\Big), \tag{37}$$

where θ is a vector of protocol or architecture parameters and β is an effective Lagrange multiplier summarizing environmental scarcity and constraints.

9.2 Biological interpretation

Equation (37) becomes biologically interpretable under an interface mapping:

- θ: heritable architecture parameters (regulatory topology, metabolic allocation, repair intensity, oscillator coupling ratios, morphology).
- $J_N(\theta)$: finite-lifetime accumulated mismatch (phase-friction certificate, or an explicitly defined estimator derived from it) under the environment and readout protocol.
- $C_{\mathrm{comp}}(\theta)$: energetic/architectural burden (geometric impedance, transport cost, memory maintenance, control latency).
- β: environmental constraint strength (resource scarcity, stress, noise level).

Natural selection can then be seen as pushing populations toward protocols/architectures that keep mismatch auditable and bounded within the finite horizon, without exceeding the dissipation budget required for maintenance. In the AEC framing, this is equivalent to improving long-run predictive efficiency η_{pred}.

10 Openness and undecidability: why life must be interactive prediction

Computational teleology emphasizes a structural boundary: in universal computational substrates (including universal QCA-like dynamics), many future properties are undecidable or infeasible to predict within bounded resources [21, 33, 34]. Therefore, biological prediction cannot be "full simulation" of the environment; it must be interactive, approximate, and continuously updated.

Online correction near the undecidability boundary. In the AEC language, a system maintains survival only if it sustains the inequality (24). When environmental novelty exceeds the compressive capacity of the internal model, the predictive information rate $\dot{I}_{\mathrm{pred}}$ drops. To restore viability, the agent has limited options:

- increase exploration/measurement effort (raising dissipation and possibly increasing $\dot{I}_{\mathrm{pred}}$),
- simplify the task by lowering effective resolution (sacrificing precision to regain predictability),
- reconfigure architecture (paying geometric Landauer costs) to support faster model updating.

These options are precisely the trade-offs seen in biological adaptation: stress responses increase metabolic costs, sensory systems adjust gain and resolution, and learning reorganizes circuits at energetic expense. These are standard features of cellular and neural adaptation mechanisms [24, 29].

Intelligence as rapid model reconfiguration. Within this view, "intelligence" is not a separate ontological entity; it is an AEC capability for rapid interactive model updates that maintain η_{pred} when prediction degrades. This provides an interface-level explanation of why living systems exhibit exploration, plasticity, and multi-scale correction: such features are forced by readout constraints and by the presence of hard computational limits on prediction.

11 Testable predictions and experimental/data-analysis protocols

The appropriate empirical posture for HPA–Ω is to specify testable interfaces: statistical biases, scaling laws, and protocol-level certificates that can be measured across systems. This section gives three minimal, falsifiable templates aligned with the AEC definition.

11.1 Audit posture: bounded-complexity protocols

Each test below depends on explicit protocol choices: how a readout variable is mapped to a phase coordinate (uniformization), how ratios and tolerances are inferred, which window length N and step size Δt are used, which smoothing operator is applied, and which surrogate/baseline family is used for comparison. To avoid post-hoc freedom, these choices should be fixed *a priori* or selected by a deterministic bounded-complexity rule from a finite admissible domain (Appendix G, Definitions G.1–G.2). All reported effects should be stated relative to a baseline that preserves the relevant sampling and noise structure of the measurement protocol.

11.2 P1: rhythm ratio bias and the anti-locking index Q_δ

Data. Cross-species or cross-condition measurements of coupled biological rhythms (e.g. heart–respiration, gait frequencies, circadian harmonics, neural band couplings), together with estimates of noise levels and coupling strengths.

Pipeline.

1. Infer an effective frequency ratio α using standard phase-synchrony or coupled-oscillator identification methods [10, 11].
2. Estimate an operational locking tolerance δ from noise/coupling (or from empirical locking width) using the detuning-to-ratio conversion (33).
3. Compute $Q_\delta(\alpha)$ as in Eq. (28) (Appendix H provides a reference implementation).
4. Compare the distribution of Q_δ (or normalized Q_δ relative to the Diophantine bound) against appropriate baselines that preserve sampling and noise structure.

Decision criterion. If there is no anti-locking selection pressure, Q_δ should not systematically exceed baselines. If the control-law hypothesis holds (Remark 7.2), Q_δ should be significantly shifted upward and should exhibit an enhanced upper tail near the golden-branch extremum.

Audit form (baseline and rejection region). Fix an admissible protocol domain (windowing, inference method, and surrogate family) and compute the scale-free score $Z_\delta = \sqrt{\delta}\, Q_\delta$ (Eq. (36)) for each inferred coupling ratio. Let T be a pre-specified summary statistic of the empirical Z_δ sample (e.g. median or an upper-tail quantile). Generate a baseline distribution of T using protocol-matched surrogates (e.g. time-shuffled or phase-randomized controls that preserve power spectra and noise levels) and reject the no-bias hypothesis at level α if T exceeds the $(1 - \alpha)$ baseline quantile.

11.3 P2: phase-friction certificate E_N vs maintenance dissipation

Claim. Stronger AEC should manifest as lower effective mismatch density σ (or lower discrepancy-based proxies such as D_N^* at matched scales) under comparable external perturbations, at the cost of higher maintenance dissipation $\dot{W}_{\text{diss}}$.

Operational proxies. Fix a readout variable $y(t)$ (e.g. inter-event intervals, phase-difference increments, or any scalar readout chosen by the protocol) and map it to a phase coordinate $u(t) \in [0, 1)$ by uniformization under a reference distribution:

$$u(t) := \widehat{F}(y(t)), \tag{38}$$

where $\widehat{F}$ is an empirically estimated CDF under matched baseline conditions. For a window of N samples ending at time t, compute the star discrepancy $D_N^*(t)$ of $\{u_{t,1}, \ldots, u_{t,N}\}$ and define

$$E_N(t) := N D_N^*(t), \qquad S_{\mathrm{pf}}^{(N)}(t) := k_{\mathrm{B}} E_N(t). \tag{39}$$

Define the external-time mismatch rate by a finite difference

$$\Sigma_N(t) := \frac{E_N(t + \Delta t) - E_N(t)}{\Delta t}, \tag{40}$$

which estimates $\Sigma(x, t)$ after smoothing (Section 3.1).

For the dissipation side, use a protocol-matched estimate of maintenance power $P_{\mathrm{maint}}(t)$ (e.g. metabolic power minus external mechanical work, or ATP expenditure rates). In the minimal closure where maintenance is dominated by exporting phase-friction entropy at temperature T_c, the bound (25) gives

$$P_{\mathrm{maint}}(t) \;\geq\; k_{\mathrm{B}} T_c \, \Sigma(x, t), \tag{41}$$

and with geometric impedance one expects an affine lower envelope

$$P_{\mathrm{maint}}(t) \;\gtrsim\; k_{\mathrm{B}} T_c \, \Sigma(x, t) \;+\; P_{\mathrm{geom}}(t), \tag{42}$$

where P_{geom} aggregates architecture-dependent overhead terms (Section 5.1).

Decision criterion. Across matched conditions, systems with enhanced repair/control should exhibit reduced Σ_N at the relevant scale. For quantitative fitting, one can regress P_{maint} against Σ_N to estimate $k_{\mathrm{B}} T_c$ as the minimal slope and diagnose additional intercept/overhead terms associated with Z_{geom}. To make this auditable, the window length N, step size Δt, and smoothing operator used to compute Σ_N should be fixed (or selected by a bounded-complexity rule) and reported alongside the fitted slope/intercept and their uncertainty under resampling.

11.4 P3: hierarchical relaxation and $1/f$ structure from Fibonacci/Zeckendorf layering

HPA–Ω predicts that canonical multi-scale coding (Ostrowski/Zeckendorf hierarchies) can imprint hierarchical time scales on readout-induced residuals [4]. If biological repair/relaxation processes aggregate across a roughly logarithmic ladder of time scales, a mid-band $1/f$ spectrum can appear as a robust template (with task- and protocol-dependent prefactors). This yields a falsifiable interface: identify the relevant band, estimate the slope and prefactor, and test consistency with hierarchical-ladder aggregation rather than with purely white-noise assumptions. For background on $1/f$ phenomenology and spectral estimation practices, see e.g. [35].

11.5 Reference implementations

Appendix H includes minimal Python reference implementations for:

- logarithmic mismatch-growth compatibility for the golden branch vs linear growth for rationals (Experiment A),
- a numerical illustration of the predictive-information-rate threshold (26) (Experiment B),
- computation of Q_δ (Experiment C).

The scripts are provided for transparent reproducibility of the certificate computations and threshold numerics used in the text.

12 Conclusion

In the HPA–Ω framework, irreversibility and entropy production are operational consequences of finite-resolution scan–projection readout rather than ontic randomness. Arithmetic statistical mechanics supplies a computable mismatch certificate (star discrepancy and accumulated mismatch) that quantifies *phase friction*. Within that interface language, this paper proposed a strict operational definition of life: *life is a predictive active error-correction (AEC) phase*—a subsystem that acquires conditional information, maintains a predictive internal model, and applies feedback that lowers its own phase-friction entropy production relative to a passive baseline while paying the required dissipation costs.

Information thermodynamics and a geometric Landauer principle yield a necessary inequality for sustained existence: predictive gain must exceed dissipation, implying a predictive information-rate threshold. This turns "biological teleology" into an engineering objective: maximize predictive efficiency under architectural constraints. We further proposed a falsifiable control-law hypothesis for adaptive rhythms: to resist low-order locking, effective coupling ratios should be biased toward badly-approximable irrationals, with the golden branch as the extremal anti-locking reference.

The result is a unified interface picture: genetics, homeostasis, learning, and evolution can be reformulated as multi-scale strategies for sustaining predictive AEC against unavoidable readout-induced phase friction.

A Appendix

B Symbols and minimal object table

- U_{scan}: unitary scan operator (ontic layer).
- V: pointer-phase unitary; (U_{scan}, V) form a Weyl pair.
- $\alpha \in (0,1) \setminus \mathbb{Q}$: scan slope for irrational rotation.
- $x_n = x_0 + n\alpha \pmod 1$: Kronecker orbit on $[0,1)$.
- D_N^*: star discrepancy of $\{x_1, \ldots, x_N\}$.
- $E_N := N D_N^*$: accumulated mismatch (audit certificate).
- $S_{\mathrm{pf}}(N) := k_{\mathrm{B}} E_N$: phase-friction entropy certificate on a length-N window.
- τ: intrinsic scan time (iteration count).
- $\sigma(x,\tau) := \frac{1}{k_{\mathrm{B}}} \frac{\mathrm{d}S_{\mathrm{pf}}(x;\tau)}{\mathrm{d}\tau}$: mismatch density (entropy production density in scan time).
- t: external time.
- $\kappa(x)$: routing overhead (implementation cost).
- $\mathcal{N}(x) := \kappa_0/\kappa(x)$: computational lapse.
- T_c: computational temperature (operational temperature at the interface).
- W_{erase}: work required for erasure/re-encoding.
- Z_{geom}: geometric impedance term in geometric Landauer principle.
- $\dot{F}_{\mathrm{pred}}$: predictive free-energy gain rate achievable via measurement–feedback control.

- $\dot{I}_{\text{pred}}$: predictive mutual information rate (internal state vs future readout).
- $\dot{W}_{\text{diss}}$: dissipation rate required for maintenance/repair against phase friction.
- $\eta_{\text{pred}} := \dot{F}_{\text{pred}}/\dot{W}_{\text{diss}}$: predictive efficiency.
- $Q_\delta(\alpha)$: resonance susceptibility / anti-locking index at tolerance δ.

C From discrepancy to phase friction: key bounds and "locking thermal death"

This appendix collects standard discrepancy facts used in Sections 2.2–2.3.

C.1 Star discrepancy and accumulated mismatch

For a point set $\{x_1, \dots, x_N\} \subset [0,1)$, the star discrepancy is

$$D_N^* = \sup_{0 \le a \le 1} \left| \frac{1}{N} \#\{x_n < a\} - a \right|. \tag{43}$$

If $y_1 \le \cdots \le y_N$ are the sorted points, one has the exact 1D formula

$$D_N^* = \max \left\{ \max_{1 \le i \le N} \left(\frac{i}{N} - y_i \right), \max_{1 \le i \le N} \left(y_i - \frac{i-1}{N} \right) \right\}. \tag{44}$$

The accumulated mismatch is $E_N := N D_N^*$ and the phase-friction entropy certificate is $S_{\text{pf}}(N) = k_{\text{B}} E_N$.

C.2 Badly-approximable slopes yield logarithmic mismatch growth

For Kronecker sequences $x_n = x_0 + n\alpha \pmod 1$, discrepancy bounds depend on Diophantine properties of α [9, 14, 19]. The main quantitative certificate used in the text is Theorem 2.2: if $\alpha = [0; a_1, a_2, \dots]$ and $q_m \le N < q_{m+1}$, then

$$E_N \le 1 + \sum_{i=1}^{m} a_i. \tag{45}$$

If $a_i \le A$ for all i, then $E_N \le 1 + A m(N)$, and since $q_m \ge F_{m+1}$ for every continued fraction (because $a_i \ge 1$), one has the explicit bound

$$m(N) \le \left\lceil \log_\varphi\!\left(\sqrt{5}\, N\right) \right\rceil, \qquad E_N \le 1 + A \left\lceil \log_\varphi\!\left(\sqrt{5}\, N\right) \right\rceil, \tag{46}$$

which makes the logarithmic mismatch-growth compatibility fully explicit. This is the controlled-mismatch regime used as the operational "non-locking" phase.

C.3 Rational slopes yield linear mismatch growth

If $\alpha = p/q$ is rational, the orbit is periodic with period q. In particular, the empirical distribution over $[0,1)$ cannot approach uniformity, and one has the universal bound

$$D_N^* \ge \frac{1}{2q}, \qquad E_N \ge \frac{N}{2q} \tag{47}$$

for infinitely many N (including multiples of q). In the phase-friction interpretation, this corresponds to an operational "thermal death" by phase locking: mismatch accumulates linearly because the readout repeatedly revisits the same finite orbit.

C.4 Golden branch as the canonical extremum

The golden branch φ^{-1} is characterized by continued fraction coefficients $a_i \equiv 1$, hence it minimizes $\sum_{i=1}^{m} a_i$ for a given index m and maximizes the rate at which q_m grows relative to that sum. Together with Hurwitz/Markov extremality for Diophantine approximation constants [9], this makes the golden branch a canonical arithmetic choice when the operational objective is to delay low-order rational resonances under finite tolerance.

D Thermodynamic bounds for predictive AEC

This appendix summarizes the inequality chain used in Section 5. We emphasize that these are *necessary* conditions at the interface level, not sufficient conditions for biological viability.

D.1 Predictive gain and mutual information rate

In measurement–feedback thermodynamics, information obtained about a system can be converted into work or free-energy advantage, but the advantage is bounded by mutual information [7, 8]. In its simplest form, one obtains inequalities of the type

$$\langle W \rangle \;\leq\; k_{\mathrm{B}} T\, I \tag{48}$$

up to sign conventions and depending on whether W denotes extracted work or required work. At the HPA–Ω interface we use the computational temperature T_c and consider a rate form:

$$\dot{F}_{\mathrm{pred}} \;\leq\; k_{\mathrm{B}} T_c\, \dot{I}_{\mathrm{pred}}. \tag{49}$$

The operational content is that predictive free-energy gain cannot exceed the energetic scale per nat times the predictive information rate (our convention $\log = \ln$). If $\dot{I}_{\mathrm{pred}}$ is expressed in bits per unit time, the corresponding bound acquires a factor $\ln 2$.

D.2 Survival inequality and predictive efficiency

Let $\dot{W}_{\mathrm{diss}}$ be the dissipation rate required to resist phase friction and to maintain internal structure and model memory (including geometric impedance contributions). A necessary condition for sustained AEC is a positive margin between predictive gain and dissipation:

$$\dot{F}_{\mathrm{pred}} > \dot{W}_{\mathrm{diss}}. \tag{50}$$

Combining the two inequalities yields the predictive information-rate threshold

$$\dot{I}_{\mathrm{pred}} > \frac{\dot{W}_{\mathrm{diss}}}{k_{\mathrm{B}} T_c}. \tag{51}$$

If the dominant maintenance burden is exporting phase-friction entropy at computational temperature T_c, then the second law implies

$$\dot{W}_{\mathrm{diss}} \geq T_c \frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}t} = k_{\mathrm{B}} T_c\, \Sigma(x,t), \tag{52}$$

and hence $\dot{I}_{\mathrm{pred}} > \Sigma(x,t)$ (Eq. (26)). This motivates the predictive efficiency

$$\eta_{\mathrm{pred}} := \frac{\dot{F}_{\mathrm{pred}}}{\dot{W}_{\mathrm{diss}}}. \tag{53}$$

E Geometric Landauer: why correction costs are architecture-dependent

Landauer's principle bounds the minimal dissipation for logically irreversible operations [5, 6]. HPA–Ω refines this statement by emphasizing that operational information processing happens on constrained architectures (locality, routing, finite signal speed) and therefore incurs an additional geometric impedance cost [4, 21]:

$$W_{\text{erase}} \;\geq\; k_{\text{B}} T_c \ln 2 + Z_{\text{geom}}. \tag{54}$$

The term Z_{geom} is not universal; it depends on the physical organization of the information-processing substrate.

E.1 Biological proxies for geometric impedance

In biological systems, candidate proxies for Z_{geom} include (non-exhaustively):

- molecular transport distances and congestion (diffusion vs active transport),
- network topology and path-length distributions in signaling and regulatory networks,
- sparsity and long-range wiring costs in neural circuits,
- parallelism limits in repair pathways (bottlenecks and queueing),
- spatial localization constraints for assembly and proofreading operations.

Operationally, one can treat Z_{geom} as a fitted impedance term in an energy budget for correction tasks and then test whether its fitted variation correlates with measurable architectural features.

F Statistical test details for the golden-branch control-law hypothesis

Given samples $\{(\alpha_i, \delta_i)\}_{i=1}^{M}$ inferred from data (Section 11.2), define

$$Q_{\delta_i}(\alpha_i) := \min\left\{ q \in \mathbb{N} : \; \exists p \in \mathbb{Z} \text{ s.t. } \left|\alpha_i - \frac{p}{q}\right| < \delta_i \right\}. \tag{55}$$

The following statistical choices are natural:

- **Matched baselines.** Construct baselines by randomizing phases or shuffling within comparable noise/coupling strata so that δ_i and sampling protocols are preserved.
- **Normalization.** Normalize $Q_{\delta_i}(\alpha_i)$ by the Diophantine lower bound scale $q_{\min}(\delta_i) \approx (c/\delta_i)^{1/2}$ to compare across different tolerances.
- **Tail sensitivity.** In addition to mean shifts, test upper-tail enhancement (e.g. quantiles) because the hypothesis predicts increased anti-locking robustness.

F.1 Computing Q_δ in practice

The definition of Q_δ is constructive: for each q one only needs to check whether there exists an integer p such that $|\alpha - p/q| < \delta$. A practical computation is to set $p = \lfloor \alpha q \rceil$ (nearest integer) and check both $\lfloor \alpha q \rfloor$ and $\lceil \alpha q \rceil$. Appendix H provides a reference Python implementation.

F.2 A quantitative null baseline for Q_δ

If α is drawn uniformly from $(0,1)$, then $Q_\delta(\alpha) \le Q$ iff α lies within δ of some reduced rational p/q with $1 \le q \le Q$. By a union bound over reduced rationals, one has

$$\mathbb{P}(Q_\delta(\alpha) \le Q) \le 2\delta \sum_{q=1}^{Q} \phi(q), \tag{56}$$

and the classical summatory estimate $\sum_{q \le Q} \phi(q) = \frac{3}{\pi^2} Q^2 + O(Q \log Q)$ [30] yields the baseline scaling $\mathbb{P}(Q_\delta \le Q) \lesssim \frac{6}{\pi^2} \delta Q^2$ when $\delta Q^2 \ll 1$. This provides a quantitative reference curve for goodness-of-fit tests and tail comparisons under matched tolerances.

Scale-free normalization. Since the null scale is $Q_\delta \asymp \delta^{-1/2}$, it is natural to compare samples using the normalized score

$$Z_\delta(\alpha) := \sqrt{\delta}\, Q_\delta(\alpha), \tag{57}$$

which is $O(1)$ under the null and directly comparable across different tolerances.

G Interface isomorphisms: stable sectors, mismatch certificates, and active correction

This appendix records a shared protocol-level template in the HPA–Ω program: finite-resolution scan–projection readout induces symbolic words and coarse observables; stability/consistency constraints select a compressed visible sector; and sustained low-entropy structure requires either passive compensation (connections enforcing consistency) or active correction (feedback control reducing mismatch).

G.1 A shared interface template

We separate the discussion into the same two layers used throughout the manuscript:

- **Ontic scan layer.** Microscopic dynamics is unitary and reversible. Time is realized as scan iteration.
- **Operational readout layer.** Observables arise from finite windows and finite resolution. Discreteness and irreversibility are protocol consequences rather than ontic primitives.

Within the operational layer, a broad class of problems can be organized by the following interface objects:

- **Readout alphabet.** A finite word alphabet $\Omega_m = \{0,1\}^m$ (or a finite outcome set for a POVM-like instrument) obtained by window projection.
- **Stability/mismatch mechanism.** Either (i) explicit stability predicates/defect functions that select a stable subset $X_m \subset \Omega_m$, or (ii) computable mismatch certificates comparing finite readout statistics to an ideal reference (e.g. discrepancy-based certificates).
- **Coarse-graining and degeneracy.** Many-to-one maps from microstates to stable types (or from microscopic configurations to discrete outputs) generate degeneracy distributions that trade resolution for robustness.
- **Correction/compensation.** Consistency can be enforced passively by compensating connections (a protocol-geometric bookkeeping of local rephasing/transport) or actively by feedback that reduces mismatch relative to a passive baseline.

Interface object	Stable-sector language	AEC/biological language
finite readout alphabet	window words $w \in \Omega_m = \{0,1\}^m$	discretized outcomes from finite-resolution sensors/thresholds
stability selection	admissible/stable subset $X_m \subset \Omega_m$ defined by protocol constraints	viable operating region of the agent under implementation and readout constraints
mismatch/defect quantifier	defect predicates $D(\cdot)$ certifying protocol inconsistency	discrepancy/mismatch certificates D^*_N, E_N certifying readout bias accumulation
coarse graining	many-to-one folding $\Omega_m \twoheadrightarrow X_m$ with degeneracy	many-to-one coding (e.g. genetic degeneracy) increasing robustness under readout noise
consistency enforcement	compensating connections (protocol-local bookkeeping of transport/rephasing)	feedback control and repair redirecting dissipation into waste channels
resource accounting	implementation cost as an audit constraint (bounded-complexity closure)	Landauer-scale and architecture-dependent costs bounding sustainable correction
observable signatures	rigid finite counts/histograms and thresholded spectrum changes	statistical biases/scaling laws in Q_δ, E_N, Σ under matched protocols

Table 1: A protocol-level isomorphism dictionary: stable-sector constructions and predictive AEC can be viewed as two realizations of the same interface template (finite readout, mismatch/stability, correction, and bounded-complexity audit).

- **Audit closure under bounded complexity.** Quantitative claims are framed as deterministic selections from finite candidate families under explicit complexity bounds, together with rigidity/stabilization diagnostics.

G.2 Isomorphism dictionary (stable sectors $\leftrightarrow$ AEC)

Table 1 summarizes a protocol-level correspondence between (a) stable-sector constructions in finite-resolution readout models and (b) predictive AEC mechanisms that suppress readout-induced mismatch.

G.3 Audit template: bounded-complexity closure and rigidity

To state quantitative claims in an auditable form, we use a bounded-complexity selection principle.

Definition G.1 (Bounded-complexity closure (audit form)). *Fix reference targets $x_i^{\mathrm{ref}} > 0$ and a candidate family $x_i(\theta) > 0$ indexed by discrete parameters θ. For a bound $B \in \mathbb{N}$, let $\Theta(B)$ be a finite admissible domain (the* complexity box*). Define the log-mismatch vector*

$$e_i(\theta) := \log\left(\frac{x_i(\theta)}{x_i^{\mathrm{ref}}}\right),$$

and summary objectives

$$E_\infty(\theta) := \max_i |e_i(\theta)|, \qquad E_1(\theta) := \sum_i |e_i(\theta)|.$$

A bounded-complexity closure is the selection of a unique $\theta_B \in \Theta(B)$ by a fully specified lexicographic minimization rule (first E_∞, then E_1, then stated secondary tie-break criteria).

Definition G.2 (Rigidity certificate)**.** *A closure is called* rigid *on a tested range* $B \in \{1, \ldots, B_{\max}\}$ *if the minimizer is unique at each* B *and stabilizes: there exists* $B_* \le B_{\max}$ *such that* $\theta_B = \theta_{B_*}$ *for all* $B_* \le B \le B_{\max}$.

G.4 Transferable falsifiable problems

The interface dictionary suggests cross-domain falsifiability questions that do not rely on post-hoc freedom:

- **Degeneracy–robustness link.** Do observed many-to-one code degeneracies correlate with reduced mismatch certificates under matched protocols, at the expected energetic cost?
- **Anti-locking selection.** Under an operational tolerance δ, do inferred coupling ratios exhibit an upward shift in Q_δ relative to baselines that preserve sampling/noise structure?
- **Thresholded sector growth.** If effective window length changes with environment or scale, do stable-type counts and splits change in constrained batches dictated by the underlying grammar/stability channel?
- **Cost slopes.** Does maintenance power admit a lower-envelope slope consistent with a computational temperature scale when regressed against a protocol-matched mismatch-rate estimator?

H Reference implementations (Python)

This appendix contains reference implementations for Section 11.5. They require only Python 3 (no third-party dependencies). A minimal requirement file is provided in `requirements.txt`.

H.1 What is reproduced

The scripts in `scripts/` reproduce the following protocol-level computations used in the main text:

- **Discrepancy and accumulated mismatch.** Exact 1D star discrepancy D_N^* for a finite phase-point set and the accumulated mismatch $E_N = N D_N^*$ (Section 2).
- **Predictive-information-rate threshold (toy illustration).** A numerical demonstration of the inequality chain leading to the threshold form (26) (Section 5). This is a unit test of the interface inequalities rather than a biological model.
- **Anti-locking index.** A constructive computation of $Q_\delta(\alpha)$ together with the golden-branch Hurwitz-scale lower bound (31) (Section 7).

H.2 How to run (examples)

- `python3 scripts/experiment_a_star_discrepancy.py`
- `python3 scripts/experiment_b_predictive_threshold.py`
- `python3 scripts/experiment_c_qdelta.py`

All scripts print a small, deterministic summary to standard output.

H.3 Sanity checks and expected patterns

- **Experiment A.** For a rational slope (phase locking), the reported E_N grows approximately linearly with N; for irrational slopes, the reported ratios $E_N/\log N$ vary slowly with N (compatibility with $O(\log N)$ growth). Under the default parameters, the rational case dominates by orders of magnitude at large N.
- **Experiment B.** With the default toy settings ($k_B T_c = 1$ and $\dot{W}_{\text{diss}} = 0.15$), the script prints whether the upper bound $\dot{F}_{\text{pred}} \le k_B T_c \, \dot{I}_{\text{pred}}$ can exceed $\dot{W}_{\text{diss}}$. Increasing environmental unpredictability (larger `p_flip`) reduces the estimated information-rate proxy and eventually flips the printed `survive?` flag to `False` for bounded memory.
- **Experiment C.** For a rational ratio (e.g. 3/5), Q_δ quickly stabilizes at a small denominator once δ is sufficiently small; for the golden branch, Q_δ increases as δ decreases and should be comparable to (and bounded below by) the printed Hurwitz-scale estimate.

H.4 Experiment A: star discrepancy and accumulated mismatch E_N

```
"""
Experiment A: star discrepancy and accumulated mismatch for rotation sequences.

Pure-Python (no third-party dependencies) reference implementation.

We compare accumulated mismatch E_N = N * D_N^* for:
- an irrational slope (golden branch),
- another irrational slope (sqrt(2) - 1),
- a rational slope (1/2) as a simple phase-locking / periodic case.
"""

from __future__ import annotations

import math


def kronecker_points(alpha: float, n: int, x0: float) -> list[float]:
    """Return points x_k = (x0 + k*alpha) mod 1 for k=1..n."""
    pts: list[float] = []
    a = float(alpha)
    for k in range(1, n + 1):
        x = x0 + k * a
        pts.append(x - math.floor(x))
    return pts


def star_discrepancy_1d(points: list[float]) -> float:
    """
    1D star discrepancy:
      D*_N = sup_{a in [0,1]} | (1/N)*#{x_i < a} - a |.

    For sorted points y_i, an exact formula is:
      max_i (i/N - y_i) and max_i (y_i - (i-1)/N).
    """
    y = sorted(points)
    n = len(y)
    if n == 0:
        return 0.0
    inv = 1.0 / float(n)
    d1 = 0.0
```

```python
    d2 = 0.0
    for i, yi in enumerate(y, start=1):
        d1 = max(d1, i * inv - yi)
        d2 = max(d2, yi - (i - 1) * inv)
    return max(d1, d2)


def accumulated_mismatch(alpha: float, n: int, x0: float) -> float:
    pts = kronecker_points(alpha, n=n, x0=x0)
    d = star_discrepancy_1d(pts)
    return float(n) * d


def main() -> None:
    phi = (1.0 + math.sqrt(5.0)) / 2.0
    alpha_golden = 1.0 / phi
    alpha_sqrt2 = math.sqrt(2.0) - 1.0
    alpha_rational = 1.0 / 2.0

    x0 = 0.123456789
    ns = [100, 300, 1_000, 3_000, 10_000, 30_000]

    print("N, E_N(golden), E_N(sqrt2-1), E_N(1/2)")
    for n in ns:
        eg = accumulated_mismatch(alpha_golden, n=n, x0=x0)
        es = accumulated_mismatch(alpha_sqrt2, n=n, x0=x0)
        er = accumulated_mismatch(alpha_rational, n=n, x0=x0)
        print(f"{n:>8d}  {eg:>12.6f}  {es:>12.6f}  {er:>12.6f}")

    print("\nCompatibility check: E_N/log N (slow variation suggests O(log N))")
    print("N, Eg/logN, Es/logN, Er/logN")
    for n in ns:
        logn = math.log(float(n))
        eg = accumulated_mismatch(alpha_golden, n=n, x0=x0) / logn
        es = accumulated_mismatch(alpha_sqrt2, n=n, x0=x0) / logn
        er = accumulated_mismatch(alpha_rational, n=n, x0=x0) / logn
        print(f"{n:>8d}  {eg:>12.6f}  {es:>12.6f}  {er:>12.6f}")


if __name__ == "__main__":
    main()
```

H.5 Experiment B: a numerical illustration of the predictive-information-rate threshold

```python
"""
Experiment B: a toy demonstration of the predictive-information-rate threshold.

This is NOT a biological model. It is a unit test for the inequality chain:
  Fdot_pred <= k_B*T_c * Idot_pred
  survival needs Fdot_pred > Wdot_diss
  => Idot_pred > Wdot_diss/(k_B*T_c)

We simulate a binary environment with tunable predictability and a simple predictor
with tunable memory, then estimate a mutual-information-rate proxy from accuracy.
"""
```

```
from __future__ import annotations

import math
import random


def simulate_environment(T: int, p_flip: float, seed: int) -> list[int]:
    """
    Binary Markov environment:
      s_{t+1} = s_t XOR Bernoulli(p_flip).
    Smaller p_flip -> more predictable.
    """
    rng = random.Random(seed)
    s = 0
    out: list[int] = []
    for _ in range(T):
        out.append(s)
        if rng.random() < p_flip:
            s ^= 1
    return out


def predictor_majority_memory(seq: list[int], m: int, seed: int) -> float:
    """
    Toy predictor with memory length m:
    predict next bit as majority of last m bits; if insufficient history, guess
    ↪  random.

    Returns empirical one-step-ahead accuracy.
    """
    rng = random.Random(seed)
    T = len(seq)
    if T < 2:
        return 0.0
    correct = 0
    for t in range(T - 1):
        if m <= 0 or t < m:
            pred = rng.randint(0, 1)
        else:
            window = seq[t - m + 1 : t + 1]
            ones = sum(window)
            pred = 1 if ones > (m / 2.0) else 0
        correct += int(pred == seq[t + 1])
    return correct / float(T - 1)


def mutual_information_rate_proxy_from_accuracy(acc: float) -> float:
    """
    Proxy: treat prediction as a binary symmetric channel with crossover e=1-acc.
    Then I = 1 - H2(e) bits/step (clipped at 0).
    """
    e = max(1e-12, min(1.0 - 1e-12, 1.0 - float(acc)))
    H2 = -(e * math.log2(e) + (1.0 - e) * math.log2(1.0 - e))
    return max(0.0, 1.0 - H2)


def main() -> None:
```

```
    # Units: set k_B*T_c = 1 so the threshold is simply Idot_pred > Wdot_diss.
    kBTc = 1.0
    Wdot_diss = 0.15

    T = 50_000
    memory_list = [0, 1, 2, 4, 8, 16, 32]
    for p_flip in [0.01, 0.05, 0.10, 0.20]:
        seq = simulate_environment(T=T, p_flip=p_flip, seed=0)
        print(f"\nEnvironment p_flip={p_flip:.2f} (smaller -> more predictable)")
        for m in memory_list:
            acc = predictor_majority_memory(seq, m=m, seed=1)
            Idot = mutual_information_rate_proxy_from_accuracy(acc)
            Fdot_upper = kBTc * Idot
            survives = Fdot_upper > Wdot_diss
            print(
                f"  m={m:2d}  acc={acc:.3f}  Idot~={Idot:.3f}  "
                f"Fdot_upper~={Fdot_upper:.3f}  survive? {survives}"
            )


if __name__ == "__main__":
    main()
```

H.6 Experiment C: computing the anti-locking index Q_δ

```
"""
Experiment C: computing the resonance susceptibility / anti-locking index
↪ Q_delta(alpha).

Definition:
  Q_delta(alpha) = min{ q in N : exists p in Z s.t. |alpha - p/q| < delta }.

We implement a constructive search by scanning q and checking the nearest p.
This is a toy utility for Section 7 and Appendix (golden-branch control-law
↪ hypothesis).
"""

from __future__ import annotations

import math


def q_delta(alpha: float, delta: float, q_max: int = 200_000) -> int | None:
    """Return Q_delta(alpha) up to q_max, or None if not found within the search."""
    a = float(alpha)
    d = float(delta)
    for q in range(1, q_max + 1):
        aq = a * q
        p0 = int(round(aq))
        # Check nearest integers (robust to rounding edge cases).
        for p in (p0 - 1, p0, p0 + 1):
            if abs(a - (p / q)) < d:
                return q
    return None
```

```
def main() -> None:
    phi = (1.0 + math.sqrt(5.0)) / 2.0
    alpha_golden = 1.0 / phi
    alpha_rational = 3.0 / 5.0

    deltas = [1e-1, 5e-2, 2e-2, 1e-2, 5e-3, 2e-3, 1e-3]

    print("delta, Q_delta(golden), Q_delta(3/5)")
    for delta in deltas:
        qg = q_delta(alpha_golden, delta=delta, q_max=200_000)
        qr = q_delta(alpha_rational, delta=delta, q_max=200_000)
        print(f"{delta:>8.1e}  {str(qg):>14s}  {str(qr):>10s}")

    print("\nHurwitz lower bound scale for golden branch:
    ↪  ceil((1/(sqrt(5)*delta))^(1/2))")
    for delta in deltas:
        bound = math.ceil(math.sqrt(1.0 / (math.sqrt(5.0) * delta)))
        print(f"{delta:>8.1e}  {bound:>6d}")


if __name__ == "__main__":
    main()
```

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