Holographic Polar Dynamics: Topological Inversion of the Schwarzschild Singularity and the Phase Origin of Gravity

Haobo Ma*
AELF PTE LTD.
#14-02, Marina Bay Financial Centre Tower 1, 8 Marina Blvd, Singapore 018981

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Abstract

General Relativity, based on Riemannian geometry and continuous manifolds, encounters an essential singularity crisis in extreme gravitational fields (such as $r \to 0$ in black holes). This paper applies the **Holographic Polar Arithmetic (HPA)** established in Paper I [31] to spacetime physics, proposing a non-perturbative geometric model—**Omega Dynamics**—in which gravity emerges from a deterministic *phase mismatch* between unitary Θ-scanning and a lower-dimensional holographic readout.

In the exterior region we recover the Schwarzschild geometry and rewrite it in isotropic radius ρ, where the metric contains an explicit Cayley inversion factor $\mathcal{J}(\rho)$ and an exact inversion symmetry $\rho \mapsto \rho_h^2/\rho$. This classical Einstein–Rosen throat template is then promoted to a dynamical continuation rule in Omega Dynamics: infalling degrees of freedom are *modeled* as passing through the throat into a boundary readout channel, so that information loss is replaced (at the level of the model axioms) by a unitary re-encoding into phase correlations of the outgoing channel.

Concretely, Omega Dynamics adopts an inversion continuation axiom and a modular-scan identification for the unitary update. Under these axioms, evaporation is modeled as a coarse-grained phase readout: marginal statistics can be approximately thermal while correlations carry the information.

1 Introduction: The Collapse and Reconstruction of Geometry

1.1 The Singularity Crisis

The Schwarzschild metric, the first exact solution to Einstein's field equations, predicts its own partial failure. At $r = 2M$, the coordinate singularity suggests a breakdown of the coordinate system, which is resolved by passing to coordinates regular on the horizon (e.g. Kruskal–Szekeres) [26,50]. However, the essential singularity at $r = 0$ remains a fundamental pathology of classical General Relativity: curvature invariants diverge there (e.g. the Kretschmann scalar), signaling a genuine geometric singularity [34,39].

1.2 The Polar Paradigm

In **Paper I (Holographic Polar Arithmetic)** [31], we established that linear arithmetic fails at the quantum scale ($\rho \to 0$) and must be replaced by polar arithmetic. In this paper, we extend this paradigm to the fabric of spacetime itself. We propose, echoing 't Hooft's Cellular Automaton interpretation [45], that time is not continuous but a discrete updating process (scanning). Omega Dynamics postulates that the would-be endpoint of radial evolution is replaced by an

*Email: auric@aelf.io

inversion channel (Axiom 3.6). We validate the *exterior* sector by showing that Phase Pressure reproduces the Newtonian limit and, under standard covariant closure assumptions, closes to the Schwarzschild exterior for spherically symmetric isolated defects (Section 2.3); we then identify the horizon as the phase-locking/redshift surface (Section 3) and use the classical isotropic Einstein–Rosen throat template [13, 26, 50] as the geometric input for the Omega continuation rule.

1.3 Conventions and Minimal Axioms Used in This Paper

Units. We use geometric units $c = G = \hbar = k_B = 1$ throughout, so the Schwarzschild parameter M has dimensions of length. The areal radius is denoted by r (so spheres have area $4\pi r^2$). We reserve ρ for the *isotropic* (conformally flat) radius used to make the inversion symmetry manifest (Section 3).

Scan time. The scan operator Θ is a unitary on a Hilbert space $\mathcal{H}$, and discrete time is the iteration index $n \in \mathbb{Z}_{\geq 0}$:

$$|\Psi_n\rangle := \Theta^n |\Psi_0\rangle. \tag{1}$$

In a macroscopic continuum limit one may introduce a coarse-grained time parameter $t := n\delta$ for some fixed scan step $\delta > 0$ (cf. Assumption 5.1). A concrete minimal model (Koopman unitary for an irrational rotation) and its basic consequences are given in Paper I; see also standard references on Koopman operators and ergodic rotations [25, 27, 31].

Holographic readout and discrepancy. Classical coordinates and phases are obtained by a coarse-graining/projection of the scan orbit. The resulting mismatch is quantified by a discrepancy functional on the observed phase sequence (Section 2).

What is classical vs. what is Omega. The isotropic Schwarzschild form and its inversion identities (Section 3) are standard results in GR. Omega Dynamics enters through (i) the interpretation of discrepancy-induced mismatch as an effective force (Phase Pressure, Section 2), and (ii) the inversion continuation rule that treats the throat as a holographic readout channel (Axiom 3.6).

Axioms/assumptions used. For referee-readability, all Omega-specific inputs are explicitly marked as assumptions/axioms: mismatch sourcing (Assumption 2.7), defect Gauss law (Axiom 2.10), covariant continuum closure (Assumption 2.15), inversion continuation (Axiom 3.6), modular-scan identification (Assumption 5.1), and the algebraic inversion channel (Axiom 5.4).

2 The Holographic Origin of Gravity: Phase Pressure

2.1 Quantum Gap and Projection Discrepancy

In Paper I [31], we established that the superposition of two polar states $\Psi_A + \Psi_B$ is not perfectly closed under unitary Θ-scanning. The residual term, or "quantum gap", is not random noise but a deterministic consequence of projecting a high-dimensional helix onto a lower-dimensional grid. Rigorously, this is quantified by the **Discrepancy** D_N of the sequence of projection phases $\{\vartheta_n\}$.

Definition 2.1 (Phase readout sequence (local model))**.** *Fix a coarse-grained readout cell (spatial point)* $\mathbf{x}$*. A* holographic readout *is a map that assigns to the scan orbit* $|\Psi_n\rangle = \Theta^n|\Psi_0\rangle$ *a phase* $\vartheta_n(\mathbf{x}) \in [0, 2\pi)$*. We write the normalized phases* $x_n(\mathbf{x}) := \vartheta_n(\mathbf{x})/(2\pi) \pmod 1 \in [0, 1)$ *and suppress the explicit* $\mathbf{x}$*-dependence when no confusion can arise.*

Definition 2.2 (Star discrepancy of a phase orbit)**.** *Let* $x_n := \vartheta_n/(2\pi) \pmod 1 \in [0, 1)$ *be the normalized projection phases. The (one-dimensional) star discrepancy of the first* N *points is*

$$D_N^* := \sup_{0 \leq a \leq 1} \left| \frac{1}{N} \#\{1 \leq n \leq N : x_n < a\} - a \right|. \tag{2}$$

Remark 2.3 (Discrepancy controls coarse-grained observable bias). *In one dimension, the star discrepancy governs the worst-case bias of coarse-grained observables: for any function f of bounded variation on $[0,1)$,*

$$\left| \frac{1}{N} \sum_{n=1}^{N} f(x_n) - \int_0^1 f(x)\, dx \right| \leq V(f)\, D_N^*, \tag{3}$$

where $V(f)$ is the total variation. This is the Koksma–Hlawka inequality (in its one-dimensional form) and provides an operational reading of D_N^* as a uniform control on readout bias across a natural observable class; see, e.g., [27].

Remark 2.4 (Empirical-measure viewpoint). *For a fixed readout cell, define the empirical phase measure $\mu_N := \frac{1}{N}\sum_{n=1}^{N} \delta_{x_n}$ on $[0,1)$ and its deviation from uniform Lebesgue measure λ by $\nu_N := \mu_N - \lambda$. Then*

$$\frac{1}{N} \sum_{n=1}^{N} f(x_n) - \int_0^1 f\, d\lambda = \int_0^1 f\, d\nu_N, \tag{4}$$

and D_N^* is exactly the supremum of $|\nu_N([0,a))|$ over anchored intervals. In the macroscopic model, the role of $\sigma(\mathbf{x})$ is to summarize this readout deviation after spatial and temporal coarse-graining, while the Poisson/Dirichlet prescription can be viewed as the standard Green-function response of a local potential to a source density [20].

Weyl's equidistribution theorem implies $D_N^* \to 0$ for Kronecker orbits $x_n = \{n\alpha + \beta\}$ with $\alpha \notin \mathbb{Q}$ [27, 52]. Moreover, for *badly approximable* α (in particular for the golden slope singled out in Paper I), one has the quantitative bound

$$D_N^* = O\left(\frac{\log N}{N}\right) \tag{5}$$

with an implied constant depending on α [27].

The key point for dynamics is that although the *per-step* mismatch vanishes ($D_N^* \to 0$), the *accumulated* mismatch

$$\mathcal{E}_N := N D_N^* \tag{6}$$

can grow slowly. In particular, for badly approximable slopes (including the golden case), quantitative discrepancy estimates imply

$$\mathcal{E}_N = O(\log N), \tag{7}$$

with constants controlled by the continued-fraction data of α; see, e.g., [10, 24, 27].

Remark 2.5 (Mismatch as a maximal counting deviation). *The quantity $\mathcal{E}_N$ can be rewritten as*

$$\mathcal{E}_N = \sup_{0 \leq a \leq 1} \left| \#\{1 \leq n \leq N : x_n < a\} - Na \right|, \tag{8}$$

so it measures the largest *deviation (in absolute count) of empirical phase statistics from uniformity over all anchored intervals $[0,a)$.*

Remark 2.6 (Discrepancy as a primal impulse (interpretive layer)). *Beyond its role as a quantitative equidistribution error, discrepancy can be read as the primitive* impulse *that drives the macroscopic sector of Omega Dynamics. Because the holographic readout cannot perfectly resolve the unitary Θ-scan, the accumulated mismatch $\mathcal{E}_N$ forces the continuum description to introduce a compensating potential Φ whose Phase Pressure $-\nabla\Phi$ is experienced as gravity. In this sense, gravity functions as a "repair force" that attempts to close the geometric gap between scan dynamics and readout.*

Assumption 2.7 (Continuum phase potential sourced by mismatch). *In the macroscopic continuum limit, the holographic readout assigns to each point (or coarse-grained cell) a scalar* mismatch density $\sigma(\mathbf{x})$ *obtained by coarse-graining the microscopic discrepancy accumulation (e.g. from* $\mathcal{E}_N(\mathbf{x})$ *at large* N*). This mismatch density sources a static scalar* ***phase potential*** $\Phi(\mathbf{x})$ *through a local Poisson law*

$$\Delta\Phi = 4\pi\rho_\Phi, \qquad \rho_\Phi := \kappa_\Phi\,\sigma, \tag{9}$$

where the coupling κ_Φ *is fixed by matching to the Newtonian limit. The associated* ***Phase Pressure*** *(force per unit test mass in the Newtonian sector) is conservative and given by*

$$\mathcal{P}_\Phi := -\nabla\Phi. \tag{10}$$

Equivalently, Φ *is characterized as a stationary point of a local quadratic Dirichlet functional (Appendix A.2).*

Remark 2.8 (Normalization and calibration of the mismatch sector). *The proportionality* $\rho_\Phi = \kappa_\Phi\sigma$ *carries no independent freedom once the Newtonian limit is fixed: one may absorb* κ_Φ *into the definition of* σ *and regard* ρ_Φ *as the calibrated mismatch density. In particular, for an isolated defect the Gauss-law normalization (Axiom 2.10) fixes the total phase charge*

$$\mathcal{Q}_\Phi(V) := \int_V \rho_\Phi\, d^3x \tag{11}$$

by matching to the asymptotic Schwarzschild mass parameter M *(below), so the overall scale of the mismatch sector is determined by the same Newtonian calibration that fixes* κ_G*.*

Remark 2.9 (Coarse-graining and renormalization of mismatch). *A fully microscopic definition of* $\sigma(\mathbf{x})$ *requires specifying how the readout chooses a time window* N *and a spatial coarse-graining scale. One natural formalization is to introduce a family of coarse-grained fields* $\sigma_\ell(\mathbf{x})$ *obtained from* $\mathcal{E}_{N(\ell)}(\mathbf{x})$ *at a resolution scale* ℓ *(with* $N(\ell)\to\infty$ *as* $\ell\to 0$*) and assume* $\sigma_\ell \to \sigma$ *in a suitable weak sense. The present paper uses only the existence of the continuum field* σ *and its Poisson coupling.*

Unlike Entropic Gravity [49], which relies on thermodynamic statistics, Phase Pressure is modeled here as a deterministic response of the readout potential Φ to projection mismatch.

2.2 Mass as Topological Phase Flux

We model mass as a defect sourcing the phase potential Φ.

Axiom 2.10 (Phase Gauss law for isolated defects). *In the static sector, an isolated defect of charge* $\mathcal{Q}_{top}$ *produces a phase potential satisfying the Gauss/Poisson law*

$$\oint_{\partial V} \nabla\Phi\cdot d\mathbf{A} = 4\pi\mathcal{Q}_{top}. \tag{12}$$

Equivalently, in distributional form $\Delta\Phi = 4\pi\rho_\Phi$ *with* $\rho_\Phi = \mathcal{Q}_{top}\,\delta_0$ *(Appendix A.3). The spherically symmetric solution is* $\Phi(r) = -\mathcal{Q}_{top}/r$ *up to an additive constant (Appendix A.1).*

To match Newtonian gravity in the weak-field limit, we identify the Newtonian potential as $\phi_N := \Phi$ and fix $\mathcal{Q}_{top} = M$ by comparing $g_{tt} \approx -(1+2\phi_N)$ to the standard Schwarzschild asymptotics $g_{tt} \approx -(1-2M/r)$ [41].

Remark 2.11 (Mass parameter as an asymptotic invariant). *Equivalently,* M *is the asymptotic mass parameter of the exterior and can be characterized invariantly (for asymptotically flat spacetimes) by the corresponding conserved mass notion (ADM/Komar, depending on hypotheses). Standard references include [40, 50].*

Proposition 2.12 (Newtonian acceleration from phase potential)**.** *Interpreting Phase Pressure as force per unit test mass, the induced acceleration is* $\mathbf{a} = -\nabla\phi_N = -\nabla\Phi$*. For* $\phi_N = \Phi = -M/r$ *one obtains the Newtonian field*

$$\mathbf{a}(r) = -\frac{M}{r^2}\,\hat{\mathbf{r}}. \tag{13}$$

Remark 2.13. *If the phase is realized globally as a* $U(1)$ *connection (so that* Φ *is only locally a single-valued potential), then the defect charge can be identified with a Chern number and is quantized, exactly as in the Dirac monopole construction [11, 56]. In Omega Dynamics we adopt this quantization as a structural input for defect charges:*

$$\mathcal{Q}_{top} = n\,q_0, \qquad n \in \mathbb{Z}, \tag{14}$$

with a fundamental unit q_0 *fixed by the microscopic theory (in geometric units one may take* q_0 *at the Planck scale). Macroscopic masses correspond to large* n*; the present paper uses only the topological conservation/additivity implied by this structure.*

Remark 2.14 (Topological quantization as a bundle invariant)**.** *The above statement can be formulated cleanly in bundle language: for a* $U(1)$ *principal bundle over* S^2 *surrounding an isolated defect, the flux of the curvature two-form through the sphere represents the first Chern class and is therefore quantized. Standard expositions include [36].*

A massive object is thus a **Phase Monopole** in the static sector. It introduces a persistent lag in the Θ-scan readout, and the induced potential $\phi_N = -M/r$ reproduces the usual weak-field metric deformation

$$g_{tt} = -(1 + 2\phi_N) + O(\phi_N^2), \qquad g_{rr} = (1 - 2\phi_N) + O(\phi_N^2), \tag{15}$$

so that $g_{tt} \approx -(1 - 2M/r)$ and $g_{rr} \approx 1 + 2M/r$ in Schwarzschild areal radius [50].

2.3 Covariant Completion: From Phase Pressure to the Einstein Equation

The previous subsections fix the *Newtonian* sector: the static phase potential Φ solves a Poisson equation sourced by a defect charge, yielding the $1/r$ potential and therefore the $1/r^2$ acceleration law. To obtain a relativistic field equation, we now state the covariant closure assumptions and invoke standard uniqueness results.

Assumption 2.15 (Local covariant continuum limit)**.** *At macroscopic scales, the coarse-grained Omega dynamics admits a continuum description by a Lorentzian metric* $g_{\mu\nu}$ *and an effective stress tensor* $T_{\mu\nu}$ *encoding phase/mismatch density, such that:*

1. *the dynamics is local and diffeomorphism invariant,*
2. *the metric field equation is second order in derivatives of* $g_{\mu\nu}$*,*
3. *asymptotically isolated defects admit an asymptotically flat limit.*

Theorem 2.16 (Uniqueness of the gravitational field equation in 4D)**.** *Under the above assumptions, the most general local, symmetric, divergence-free rank-2 tensor built from* $g_{\mu\nu}$ *and at most its second derivatives is* $G_{\mu\nu} + \Lambda g_{\mu\nu}$*, where* Λ *is a constant [30]. Hence the macroscopic field equation must take the form*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_G\, T_{\mu\nu}, \tag{16}$$

for some coupling constant κ_G*.*

Remark 2.17 (Effective field theory perspective). *Assumption 2.15 restricts attention to a second-order local continuum closure. From the effective field theory viewpoint, higher-derivative curvature terms are expected to appear at sufficiently high energies but are suppressed at long distances by the UV scale; the Einstein tensor is the universal leading term in the derivative expansion [12]. The present paper focuses on this leading macroscopic sector.*

Proposition 2.18 (Fixing κ_G by the Newtonian limit). *In the weak-field, slow-motion regime, write*

$$g_{tt} = -(1 + 2\phi_N) + O(\phi_N^2), \qquad \phi_N \ll 1, \tag{17}$$

and take $T_{tt} \approx \rho_m$ for rest-mass density ρ_m. Then the tt-component reduces to the Poisson equation

$$\Delta\phi_N = \frac{\kappa_G}{2}\,\rho_m, \tag{18}$$

see Appendix A.6. Matching to $\Delta\phi_N = 4\pi\rho_m$ fixes $\kappa_G = 8\pi$ in geometric units [50].

Corollary 2.19 (Schwarzschild exterior from Phase Pressure). *Assume an isolated defect whose macroscopic exterior is vacuum and spherically symmetric. Phase Pressure fixes $\phi_N = -M/r$ and therefore the asymptotic mass parameter. Outside the defect support, $T_{\mu\nu} = 0$ and the field equation reduces to $G_{\mu\nu} = 0$ (taking $\Lambda = 0$ for asymptotically flat solutions). By Birkhoff's theorem, the exterior geometry is Schwarzschild with mass M [34, 50].*

Remark 2.20 (Thermodynamic reading). *The same conclusion is compatible with the "Einstein equation as an equation of state" viewpoint: assuming local horizon thermodynamics (Unruh temperature and area entropy) one can derive the Einstein equation from a Clausius relation [21]. In Omega Dynamics, the phase/mismatch sector supplies the relevant coarse-grained heat/entropy flux.*

3 The Omega Metric: Rewriting Schwarzschild

3.1 Classical Schwarzschild Metric

The standard Schwarzschild metric in Schwarzschild coordinates (t, r, θ, ϕ) is given by:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{19}$$

This metric, derived by Schwarzschild in 1916 [41], exhibits a coordinate singularity at the event horizon $r_s = 2M$ and an essential singularity at $r = 0$. While the horizon singularity can be removed by coordinate transformations such as Kruskal-Szekeres coordinates [26], the curvature singularity at the center remains a non-removable feature in classical General Relativity [39].

3.2 Isotropic (Holographic) Radius and the Cayley Factor

For holographic polar dynamics it is convenient to work with the *isotropic* radius ρ, in which the spatial metric is conformally flat. The classical Schwarzschild solution admits the standard isotropic coordinate transformation [34, 50]:

$$r = \rho\left(1 + \frac{M}{2\rho}\right)^2. \tag{20}$$

In these coordinates the metric takes the form

$$ds^2 = -\left(\frac{1 - \frac{M}{2\rho}}{1 + \frac{M}{2\rho}}\right)^2 dt^2 + \left(1 + \frac{M}{2\rho}\right)^4 \left(d\rho^2 + \rho^2 d\Omega^2\right). \tag{21}$$

Define the **Holographic Inversion Factor** (a Cayley transform)

$$\mathcal{J}(\rho) := \frac{1-\frac{M}{2\rho}}{1+\frac{M}{2\rho}} = \frac{\rho-\rho_h}{\rho+\rho_h}, \qquad \rho_h := \frac{M}{2}, \tag{22}$$

so that $g_{tt} = -\mathcal{J}(\rho)^2$. The horizon $r_s = 2M$ corresponds to the finite isotropic radius $\rho = \rho_h$, where $g_{tt} \to 0$ but the spatial conformal factor $\left(1+\frac{M}{2\rho}\right)^4$ remains finite.

3.3 Inversion Symmetry and the Einstein–Rosen Throat

The isotropic form exhibits a simple inversion symmetry. Define the inversion map

$$\mathcal{I} : \rho \mapsto \frac{\rho_h^2}{\rho}. \tag{23}$$

Lemma 3.1 (Isotropic inversion identities)**.** *One has* $r(\rho) = r(\mathcal{I}(\rho))$ *and* $\mathcal{J}(\mathcal{I}(\rho)) = -\mathcal{J}(\rho)$.

Proof. Using $r(\rho) = \rho(1+\rho_h/\rho)^2 = \rho + 2\rho_h + \rho_h^2/\rho$ shows $r(\rho) = r(\rho_h^2/\rho)$. For $\mathcal{J}(\rho) = (\rho - \rho_h)/(\rho+\rho_h)$, substitution gives $\mathcal{J}(\rho_h^2/\rho) = (\rho_h - \rho)/(\rho_h + \rho) = -\mathcal{J}(\rho)$. □

In particular, $r(\rho) = \rho + 2\rho_h + \rho_h^2/\rho$ shows $r(\rho) \to \infty$ as $\rho \to 0$, so $\rho \to 0$ corresponds to a second asymptotic region in the time-symmetric isotropic completion.

Lemma 3.2 (Inversion preserves the isotropic metric form)**.** *Let* $\tilde{\rho} := \rho_h^2/\rho$*. Then*

$$\left(1+\frac{\rho_h}{\rho}\right)^4 \left(d\rho^2 + \rho^2 d\Omega^2\right) = \left(1+\frac{\rho_h}{\tilde{\rho}}\right)^4 \left(d\tilde{\rho}^2 + \tilde{\rho}^2 d\Omega^2\right), \tag{24}$$

and $g_{tt}(\rho) = g_{tt}(\tilde{\rho}) = -\mathcal{J}(\rho)^2$.

Proof. With $\rho = \rho_h^2/\tilde{\rho}$ one has $d\rho = -(\rho_h^2/\tilde{\rho}^2)\, d\tilde{\rho}$ and $\rho^2 = \rho_h^4/\tilde{\rho}^2$, hence

$$d\rho^2 + \rho^2 d\Omega^2 = \frac{\rho_h^4}{\tilde{\rho}^4} \left(d\tilde{\rho}^2 + \tilde{\rho}^2 d\Omega^2\right). \tag{25}$$

Also $(1+\rho_h/\rho)^4 = (1+\tilde{\rho}/\rho_h)^4 = (\tilde{\rho}+\rho_h)^4/\rho_h^4$, so multiplying gives the claimed identity. Finally, $\mathcal{J}(\tilde{\rho}) = -\mathcal{J}(\rho)$ by the previous lemma, hence $g_{tt} = -\mathcal{J}^2$ is invariant. □

Consequently, the spatial slice in isotropic coordinates has two asymptotically flat ends: $\rho \to \infty$ and $\rho \to 0$ correspond to $r \to \infty$ on two sheets, glued at the minimal surface $\rho = \rho_h$ (the Einstein–Rosen throat) [13, 26]. This is precisely the geometric template that Omega Dynamics promotes to a holographic "South Pole / North Pole" completion: the inversion exchanges the near-origin polar chart with a boundary chart without introducing a curvature blow-up.

Proposition 3.3 (Lower bound on areal radius in the isotropic exterior)**.** *For* $\rho > 0$*, the areal radius* $r(\rho) = \rho(1 + M/(2\rho))^2$ *satisfies*

$$r(\rho) \geq 2M, \tag{26}$$

with equality at $\rho = \rho_h = M/2$.

Proof. Writing $r(\rho) = \rho + 2\rho_h + \rho_h^2/\rho$, AM–GM gives $\rho + \rho_h^2/\rho \geq 2\rho_h$, hence $r(\rho) \geq 4\rho_h = 2M$. □

Corollary 3.4 (No curvature blow-up in the completed exterior). *In Schwarzschild vacuum the Kretschmann scalar is $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 48M^2/r^6$ [34]. On the isotropic exterior (hence also on its Einstein–Rosen completion) one has $r \geq 2M$, so*

$$K \leq \frac{48M^2}{(2M)^6} = \frac{3}{4M^4}, \tag{27}$$

and there is no $r \to 0$ curvature divergence in this geometry.

Remark 3.5 (Distinguishing the exterior throat from the classical interior). *The maximal analytic extension of Schwarzschild contains black-hole/white-hole regions and a genuine curvature singularity at $r = 0$ [26, 50]. The isotropic chart used here captures the exterior geometry in a form where the time-symmetric spatial slice exhibits an Einstein–Rosen throat. Omega Dynamics uses this throat/inversion structure as a* template *for its continuation rule, rather than as a claim that classical GR already removes the interior singularity.*

3.4 Omega Completion as a Dynamical Identification

Classically, the isotropic chart does not include the $r < 2M$ black-hole interior; it instead makes explicit the two-ended Einstein–Rosen completion of the exterior geometry. Omega Dynamics adopts the inversion $\mathcal{I}$ as the dynamical continuation rule for the holographic radius: the scan evolution in ρ is extended through $\rho = \rho_h$ by the identification $\rho \sim \rho_h^2/\rho$, so that trajectories avoid terminating at a coordinate endpoint. This provides a concrete and externally motivated realization of the "topological inversion" principle used in the remainder of the paper.

Axiom 3.6 (Omega inversion continuation). *Physical radial evolution is formulated in the isotropic holographic radius ρ. When the scan dynamics reaches the locking surface $\rho = \rho_h$, continuation is defined by switching charts via the inversion $\rho \mapsto \rho_h^2/\rho$. The inverted end $\rho \to 0$ is interpreted as a holographic boundary readout channel rather than as a curvature endpoint.*

This axiom is stated at the level of the scan-time slicing and the isotropic throat template; a fully causal Lorentzian continuation (beyond the time-symmetric exterior completion) is formulated as Conjecture 6.2 together with explicit verification criteria.

Remark 3.7 (Horizon as a viewpoint flip (semantic reading)). *Axiom 3.6 treats the locking surface not as a terminal wall but as a chart-transition interface: for an exterior observer it coincides with the black-hole horizon, while for the holographic system it plays the role of a "pupil" that redirects interior (hidden) degrees of freedom into the boundary readout channel. Under this interpretive layer, the inversion acts as a perspective flip between "inside" (latent/semantic state—and, if one wishes to read it this way, an internal conscious state) and "boundary" (record/measurement), with interior data reappearing on the boundary as phase correlations.*

4 Anatomy of a Holographic Black Hole

4.1 The Event Horizon: Phase Locking Radius

In the classical picture, the event horizon at $r_s = 2M$ is a null surface from which light cannot escape. In Omega Dynamics, we interpret this via the scanning operator Θ. In the isotropic holographic radius ρ (Section 3), the horizon corresponds to the finite **Phase Locking Radius** $\rho_{lock} := \rho_h = M/2$, characterized by the vanishing of the redshift (Cayley) factor $\mathcal{J}(\rho)$:

$$\rho = \rho_{lock} \iff \mathcal{J}(\rho) = 0 \iff g_{tt} = 0. \tag{28}$$

At this locking surface, the local proper-time tick rate of a static observer satisfies

$$\frac{d\tau}{dt} = \sqrt{-g_{tt}} = |\mathcal{J}(\rho)| \to 0 \qquad (\rho \to \rho_{lock}), \tag{29}$$

so external time t corresponds to vanishing local proper time $d\tau$: signals emitted from increasingly near the horizon are increasingly redshifted as seen by a static observer at infinity [50]. The thermal side of the same barrier is consistent with the **Unruh Effect** [48] and its black-hole specialization (Hawking temperature), with temperature proportional to surface gravity [3, 16, 17, 51]. Independently, black holes saturate the Bekenstein entropy bound / holographic scaling [2, 43, 44].

$$T_H = \frac{\kappa_{\rm sg}}{2\pi} = \frac{1}{8\pi M} \qquad \text{(Schwarzschild)}, \tag{30}$$

where $\kappa_{\rm sg}$ is the surface gravity (for Schwarzschild, $\kappa_{\rm sg} = 1/(4M)$) [50]. The corresponding Bekenstein–Hawking entropy is [2, 7, 17]

$$S_{\rm BH} = \frac{A}{4}, \qquad A = 4\pi r_s^2 = 16\pi M^2. \tag{31}$$

4.2 Near-Horizon Limit: Rindler Form in Isotropic Radius

The isotropic form makes the near-horizon limit particularly transparent. Define the (outward) proper radial distance ℓ from the locking surface by

$$d\ell := \left(1 + \frac{\rho_h}{\rho}\right)^2 d\rho, \qquad \rho_h = \frac{M}{2}. \tag{32}$$

Then $\ell = 0$ at $\rho = \rho_h$ and, for ρ close to ρ_h,

$$\ell = 4(\rho - \rho_h) + O\big((\rho - \rho_h)^2\big), \qquad \mathcal{J}(\rho) = \frac{\rho - \rho_h}{\rho + \rho_h} = \frac{\ell}{4M} + O(\ell^2). \tag{33}$$

Proposition 4.1 (Rindler $\times S^2$ near the locking surface). *In the Schwarzschild exterior rewritten in isotropic radius, the metric admits the near-horizon expansion*

$$ds^2 = -(\kappa_{\rm sg}\ell)^2\, dt^2 + d\ell^2 + (2M)^2\, d\Omega^2 + O(\ell^2), \qquad \kappa_{\rm sg} = \frac{1}{4M}. \tag{34}$$

Proof. From the isotropic form (Section 3),

$$ds^2 = -\mathcal{J}(\rho)^2\, dt^2 + \left(1 + \frac{\rho_h}{\rho}\right)^4 \left(d\rho^2 + \rho^2 d\Omega^2\right), \qquad \rho_h = \frac{M}{2}. \tag{35}$$

By definition of ℓ, one has the exact identity $(1 + \rho_h/\rho)^4\, d\rho^2 = d\ell^2$. Write $\rho = \rho_h + \delta$. Then

$$\mathcal{J}(\rho) = \frac{\rho - \rho_h}{\rho + \rho_h} = \frac{\delta}{2\rho_h} + O(\delta^2) = \frac{\delta}{M} + O(\delta^2), \qquad \ell = 4\delta + O(\delta^2), \tag{36}$$

so $\mathcal{J}(\rho) = \ell/(4M) + O(\ell^2)$ and therefore $g_{tt} = -(\kappa_{\rm sg}\ell)^2 + O(\ell^3)$ with $\kappa_{\rm sg} = 1/(4M)$. Finally, the angular factor satisfies

$$\left(1 + \frac{\rho_h}{\rho}\right)^4 \rho^2 = \frac{(\rho + \rho_h)^4}{\rho^2} = (2M)^2 + O(\delta^2) = (2M)^2 + O(\ell^2), \tag{37}$$

since the first variation vanishes at the minimal surface $\rho = \rho_h$. Combining the tt, radial, and angular pieces yields the stated expansion. □

This is the standard Rindler form (in the (t, ℓ)-plane) times a sphere of radius $2M$, and it makes the link between redshift, surface gravity, and Hawking temperature manifest: $T_H = \kappa_{\rm sg}/(2\pi)$ [3, 16, 17, 48, 50, 51].

4.3 The Interior: A Topological Inverter

In classical GR, extending through the horizon requires a coordinate choice adapted to the null surface (e.g. Kruskal coordinates). In Omega Dynamics we instead *define* the continuation through the locking surface $\rho = \rho_{lock}$ by the inversion map of the isotropic exterior geometry (Axiom 3.6). In this sense, the "interior channel" is represented by the inverted chart $\rho < \rho_{lock}$ (the second asymptotic end of the Einstein–Rosen completion on the time-symmetric slice). The coordinate ρ does not simply count down to a point; instead it inverts by $\mathcal{I}$:

$$\mathcal{I} : \rho \to \frac{\rho_{lock}^2}{\rho} \tag{38}$$

This is precisely the isotropic inversion symmetry of the Schwarzschild exterior (Section 3). The geometry is conformal to the exterior but with inverted polarity: $\rho \to 0$ corresponds to a second asymptotically flat boundary rather than a curvature blow-up. Omega Dynamics interprets this inversion as the dynamical continuation of infalling degrees of freedom into a boundary readout channel, realizing the holographic principle [32, 43] not just as a boundary condition but as an explicit map. This is an axiom-level modeling step rather than a statement about classical GR.

Remark 4.2 (Log-polar coordinate and inversion parity)**.** *Introducing the log-polar radius*

$$u := \log \frac{\rho}{\rho_h}, \tag{39}$$

the inversion symmetry becomes the simple parity map $u \mapsto -u$, with the locking surface as the fixed point $u = 0$. This makes the "topological inverter" interpretation literal: continuation across the throat is a reflection in the polar logarithmic coordinate.

4.4 The Throat and the Second Asymptotic Boundary

In the isotropic completion, the minimal surface at $\rho = \rho_h$ is the **Topological Throat** (the Einstein–Rosen bridge) connecting two asymptotic ends $\rho \to \infty$ and $\rho \to 0$ [13, 26]. In Omega Dynamics, this throat mediates the map between bulk infall and boundary degrees of freedom. This geometric channel is compatible with the **ER=EPR** conjecture [33], in which entanglement between bulk interior and boundary radiation is encoded by wormhole-like connectivity.

Remark 4.3. *We do not claim to derive ER=EPR from the Omega axioms; we only note that the "throat + boundary channel" structure provides a suggestive template for encoding correlations across the bulk/readout split. In the algebraic template of Section 5, the inversion/readout map is modeled by an antiunitary J_{inv} in the spirit of modular conjugation (Axiom 5.4). From this angle, the Einstein–Rosen throat can be viewed as a geometric avatar of a modular transformation that enables a nonlocal bulk–boundary exchange—an "interaction wormhole" at the level of the Omega model.*

4.5 Membrane Readout and Scrambling Time (Phenomenological Layer)

In the membrane paradigm, the event horizon may be replaced (for exterior observables) by an effective stretched membrane with dissipative transport properties [47]. Omega Dynamics adopts a different organizing picture—a readout interface in which the boundary channel stores information in phase correlations—but the basic lesson is compatible: exterior observers interact with an effective surface degree of freedom rather than a geometric endpoint.

Assuming the dynamics is sufficiently scrambling at the horizon scale, one expects information to be rapidly delocalized on the readout degrees of freedom. A canonical timescale is the fast-scrambling estimate [18, 42]

$$t_{\text{scr}} \sim \frac{1}{2\pi T_H} \log S_{\text{BH}} \sim 4M \log(4\pi M^2), \tag{40}$$

in geometric units (up to factors of order one). In Omega Dynamics this timescale sets the onset for the emergence of long-range phase correlations in the outgoing readout.

5 Resolution of the Information Paradox

5.1 Unitary Scanning and Modular Flow

The central claim of Paper I [31] is that the **Omega Operator** Θ is fundamentally unitary (scan time is an honest unitary evolution). A natural bridge to continuum QFT is provided by Tomita–Takesaki theory: for a von Neumann algebra $\mathcal{A}$ acting on a Hilbert space $\mathcal{H}$ and a cyclic separating vector $|\Omega\rangle$, there exists a modular operator Δ_Ω and modular conjugation J_Ω such that the modular flow is implemented by the unitary one-parameter group Δ_Ω^{it} [6, 46, 55].

In algebraic QFT, modular data are not merely formal: for the vacuum state and a Rindler wedge algebra, the modular automorphism group is precisely the Lorentz-boost flow and the state satisfies a KMS condition at the Unruh temperature; moreover, modular conjugation implements a wedge reflection (TCP) and exchanges the wedge algebra with its commutant. These statements are the content of the Bisognano–Wichmann theorem and its standard expositions [4, 5, 15].

Assumption 5.1 (Modular-scan identification)**.** *Omega scanning is modeled as a* discretization *of modular flow: for some choice of* $(\mathcal{A}, |\Omega\rangle)$ *and a fixed scan step* $\delta > 0$*,*

$$\Theta \;\equiv\; \Delta_\Omega^{i\delta}. \tag{41}$$

This assumption is motivated by the thermal-time interpretation of modular flow [9].

Remark 5.2 (Fixing the modular-time normalization)**.** *The modular parameter is not arbitrary once a concrete pair* $(\mathcal{A}, |\Omega\rangle)$ *is specified. In the Bisognano–Wichmann setting, the modular group for a Rindler wedge is the Lorentz-boost flow with a canonical* 2π *normalization, and the vacuum is a KMS state with respect to that flow [4, 5, 15]. In Omega Dynamics, this provides a quantitative calibration principle for the scan step* δ*: in the near-horizon Rindler regime (Section 4), matching the modular/KMS temperature to* $T_H = \kappa_{\rm sg}/(2\pi)$ *fixes the physical time normalization of the discrete scan.*

The second modular object, J_Ω, is an *antiunitary* involution satisfying $J_\Omega \mathcal{A} J_\Omega = \mathcal{A}'$ (the commutant) [46]. We use this structure to represent the holographic inversion channel as an antiunitary map exchanging bulk/interior and boundary readout algebras:

$$|\Psi_{\rm out}\rangle \;=\; J_{\rm inv}\, |\Psi_{\rm in}\rangle, \tag{42}$$

with $J_{\rm inv}$ playing the role of modular conjugation in the scan Hilbert space. This is compatible with the **Black Holes as Mirrors** picture [18]: once the dynamics is unitary and sufficiently scrambling [42], information is not destroyed but redistributed into subtle correlations.

Remark 5.3 (Why antiunitary is a natural inversion template)**.** *Antiunitary maps preserve transition probabilities and therefore do not entail information loss at the level of state distinguishability. More generally, Wigner's theorem characterizes probability-preserving symmetries of quantum states as unitary or antiunitary [53]; this is the mathematical reason an antiunitary involution is a natural template for an "inversion" channel that exchanges complementary observable algebras.*

Axiom 5.4 (Algebraic inversion channel)**.**

There exist von Neumann algebras $\mathcal{A}_{\rm bulk}$ *(infalling/bulk) and* $\mathcal{A}_{\rm bdry}$ *(boundary readout) acting on* $\mathcal{H}$*.*

There exists an antiunitary involution $J_{\rm inv}$ *such that*

$$J_{\rm inv}\, \mathcal{A}_{\rm bulk}\, J_{\rm inv} = \mathcal{A}_{\rm bdry}, \qquad J_{\rm inv}^2 = \mathbf{1}. \tag{43}$$

In the Tomita–Takesaki template, one may take $\mathcal{A}_{\rm bdry} = \mathcal{A}'_{\rm bulk}$ *and* $J_{\rm inv} = J_\Omega$ *for a suitable cyclic separating* $|\Omega\rangle$ *[46].*

5.2 Hawking Radiation as Decrypted Phase

Hawking radiation [17] is traditionally modeled as approximately thermal, leading to the information loss paradox if the evaporation map is taken to be fundamentally non-unitary. In Omega Dynamics, the outgoing data are modeled as a **coarse-grained readout channel** acting on the boundary phase sequence produced by unitary scanning together with the inversion channel (Axiom 5.4). In this paper we do not attempt to derive the detailed semiclassical spectrum; instead we exhibit a minimal deterministic coding mechanism in which marginal statistics can look featureless while correlations remain structured.

Remark 5.5 (Semiclassical target: graybody-corrected thermality). *In semiclassical QFT on a Schwarzschild background, the spectrum measured at infinity is thermal only up to transmission (graybody) factors. A standard mode decomposition yields number and energy fluxes of the schematic form*

$$\frac{d^2N}{dt\,d\omega} = \sum_{s,\ell,m} \frac{\Gamma_{s\ell}(\omega; M)}{2\pi} \frac{1}{\exp(\omega/T_H) \mp 1}, \qquad \frac{d^2E}{dt\,d\omega} = \omega\,\frac{d^2N}{dt\,d\omega}, \tag{44}$$

where $T_H = 1/(8\pi M)$ is the Hawking temperature (Section 4), $\Gamma_{s\ell}(\omega; M) \in [0, 1]$ are graybody factors determined by the exterior scattering potential, and the sign is $-$ for bosons and $+$ for fermions. Page computed the graybody-corrected emission rates for massless fields from a Schwarzschild black hole, providing quantitative benchmarks for fluxes and the M^{-2} scaling of the total power [37]; see also standard treatments in [3, 51].

In Omega Dynamics, this semiclassical spectrum is treated as a quantitative matching target for the coarse-grained readout layer: the thermal factor governs marginals, while information resides in correlations enforced by the unitary scan and the inversion channel.

Definition 5.6 (A minimal phase-to-radiation readout channel). *Let $x_n := \vartheta_n/(2\pi) \pmod 1 \in [0, 1)$ be the normalized scan phase. Define a binary radiation symbol by a two-interval coarse-graining,*

$$s_n := \begin{cases} 0, & x_n \in [0, 1-\alpha), \\ 1, & x_n \in [1-\alpha, 1), \end{cases} \tag{45}$$

where $\alpha \in (0, 1)$ is the irrational scan slope (Paper I). This is the standard mechanical-word coding of an irrational rotation [28, 29]. We refer to Paper I for the concrete choice of α (e.g. the golden slope) and the associated Koopman-unitary model [31].

Proposition 5.7 (Sturmian structure of the readout). *If $\alpha \notin \mathbb{Q}$ then (s_n) is Sturmian (equivalently, it has minimal subword complexity among aperiodic binary sequences) [29, 35]; see Appendix A.4.*

5.3 Thermality in Marginals vs. Information in Correlations

The symbolic readout model cleanly separates coarse one-point statistics and fine multi-time correlations.

Proposition 5.8 (Single-symbol statistics from equidistribution). *In the irrational-rotation model $x_n = \{n\alpha + \beta\}$ with $\alpha \notin \mathbb{Q}$, the binary readout defined by the window $W = [1-\alpha, 1)$ satisfies*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} s_n = \mu(W) = \alpha, \tag{46}$$

where μ is Lebesgue measure on $[0, 1)$. Equivalently, the frequency of the symbol 1 converges to α [52].

Corollary 5.9 (Finite-N deviation controlled by discrepancy). *Let D_N^* denote the star discrepancy of the first N phases $x_0, \dots, x_{N-1}$ (Section 2). Then for every N,*

$$\left| \frac{1}{N} \sum_{n=0}^{N-1} s_n - \alpha \right| = \left| \frac{1}{N} \#\{0 \le n \le N-1 : x_n < 1-\alpha\} - (1-\alpha) \right| \le D_N^*. \tag{47}$$

In particular, for badly approximable α (including the golden slope emphasized in Paper I), one has $D_N^* = O(\log N/N)$ and therefore the deviation of one-point marginals from the limiting "thermal" frequency decays at most logarithmically over N [10, 24, 27].

Proposition 5.10 (Low complexity and vanishing entropy rate of Sturmian readouts). *If (s_n) is Sturmian, its factor complexity satisfies $p(m) = m+1$ [29, 35]. Moreover, Sturmian subshifts are minimal and uniquely ergodic, hence admit a unique shift-invariant probability measure μ [22, 28]. Let $P_m(w)$ denote the μ-frequency of length-m words $w \in \{0,1\}^m$. Then the block Shannon entropy*

$$H_m := - \sum_{w \in \{0,1\}^m} P_m(w) \, \log P_m(w) \tag{48}$$

satisfies

$$H_m \le \log(m+1), \tag{49}$$

and therefore the entropy rate $\limsup_{m\to\infty} H_m/m$ vanishes.

Proof. At most $p(m) = m+1$ distinct words of length m occur in a Sturmian sequence, hence at most $m+1$ words have positive μ-frequency. Any probability distribution supported on at most $m+1$ points has Shannon entropy bounded by $\log(m+1)$, which proves the stated inequality. Dividing by m yields zero entropy rate. □

The first proposition supplies a simple mechanism for thermal-looking *marginals* under coarse-graining. The second shows that the readout remains globally constrained: while the one-point Shannon entropy $H_1 = -(1-\alpha)\log(1-\alpha) - \alpha \log \alpha$ is positive, the entropy *rate* is zero, so information can be stored in long-range correlations rather than in one-point statistics.

Remark 5.11 (A quantitative correlation signature). *The Sturmian bound $H_m \le \log(m+1)$ provides a falsifiable discriminator: for a genuinely random (memoryless) thermal readout with the same one-point frequency α, one would instead expect $H_m \approx m\, H_1$ (linear growth in block length). Thus, beyond matching one-point thermality, Omega Dynamics predicts that sufficiently fine-grained multi-time statistics of the outgoing channel should exhibit strongly sublinear complexity, with information manifesting in long-range correlation constraints rather than in marginal randomness.*

Remark 5.12 (Deterministic coding vs. stochastic readout). *The mechanical-word coding is a deterministic factor of an irrational rotation and therefore has zero entropy rate, even though its one-point entropy is positive. In Paper I [31], the observable layer involves a lossy projection readout (instrument/POVM structure) that can introduce genuine stochasticity under coarse-graining; the present subsection isolates the logical point that thermal-looking marginals do not imply information loss when information is stored in correlations.*

5.4 Decoding Template: Canonical Coding from Paper I

Paper I [31] develops a self-contained scan–projection model together with canonical integer-time coordinates (Ostrowski/Zeckendorf in the golden case) and orbit-calculus tools (orbit traces and finite-part prescriptions). In the present context, these constructions provide an explicit arithmetic template for how information can reside in the correlation structure of an apparently featureless readout: decoding is not performed from single-symbol frequencies but from the constrained combinatorics of long words and their canonical block decompositions.

Finally, in random-unitary/scrambling models of evaporation, Page's theorem implies an entropy turnover at the Page time for typical pure states [18, 38]. In Omega Dynamics, the corresponding turnover is attributed to the emergence of long-range phase correlations induced by the inversion channel J_{inv}, rather than to fundamental non-unitarity. In this sense, the black hole acts as a **holographic storage channel**: coarse-grained observables are approximately thermal, while fine-grained correlations carry the information.

6 Conclusion: The Geometric Renaissance

We have presented **Omega Dynamics**, a new framework for quantum gravity rooted in Holographic Polar Arithmetic [31]. In this framework, the classical $r = 0$ curvature endpoint is not taken as a physical spacetime point: the Omega continuation rule replaces would-be radial termination by an inversion channel built on the isotropic Einstein–Rosen throat template.

Our key findings are:

1. **Gravity as Phase Pressure:** In the continuum limit, the phase potential Φ and its associated pressure field reproduce the Newtonian sector and close to the Schwarzschild exterior under standard uniqueness assumptions [30, 34, 50].

2. **Singular Endpoints are Avoided by Inversion:** In isotropic holographic radius ρ, the Schwarzschild exterior exhibits an exact inversion symmetry $\rho \mapsto \rho_h^2/\rho$ and a minimal-surface throat at $\rho = \rho_h$ [13, 26, 50]. Omega Dynamics promotes this inversion to a continuation rule, replacing coordinate endpoints by a dual boundary channel.

3. **Information is Conserved (Model Level):** Under the modular-scan and readout assumptions, evaporation is modeled as a unitary re-encoding of infalling data into phase correlations of the outgoing channel, consistent with Page-type entanglement turnover [18,38].

Viewed through the same lens, the throat functions as an interaction wormhole (bulk $\leftrightarrow$ boundary readout), and discrepancy plays the role of a primal impulse: the mismatch between unitary scanning and holographic readout is precisely what sources the phase potential and its gravitational Phase Pressure.

These results give a concrete mathematical template (isotropic throat + inversion continuation) for replacing endpoint pathologies by a holographic continuation channel in Omega Dynamics. A next step is to extend the continuation rule from the time-symmetric slice to a fully causal spacetime description and to connect the symbolic readout model to quantitative semiclassical observables.

Quantitative Anchors and Parameter Fixing

The exterior sector has a rigid quantitative normalization:

- **Newtonian calibration.** The phase potential is identified with the Newtonian potential, $\phi_N := \Phi$, so that a point defect yields $\Phi(r) = -M/r$ and $\mathbf{a}(r) = -Mr^{-2}\hat{\mathbf{r}}$ (Section 2). This fixes the overall coupling $\kappa_G = 8\pi$ (Appendix A.6) and simultaneously calibrates the mismatch sector normalization (Remark after Assumption 2.7).

- **Classical tests in the exterior.** Since the macroscopic exterior is Schwarzschild (Corollary in Section 2.3), all standard weak-field observables follow with no additional parameters beyond the mass M; see, e.g., [54]. Restoring physical units (Appendix A.7) and writing m for the physical mass, the standard leading-order predictions include:

$$\Delta\varphi_{\text{peri}} = \frac{6\pi G m}{a(1-e^2)c^2} \quad \text{(perihelion advance per orbit)}, \tag{50}$$

$$\delta\theta_{\text{light}} = \frac{4Gm}{bc^2} \quad \text{(light deflection at impact parameter } b\text{)}, \tag{51}$$

$$\Delta t_{\text{Shapiro}} \sim \frac{2Gm}{c^3} \log\left(\frac{4r_E r_R}{b^2}\right) \quad \text{(Shapiro delay; geometry-dependent)}, \tag{52}$$

and the gravitational redshift factor $(1 - 2Gm/(rc^2))^{-1/2} - 1$ for a static emitter at radius r.

- **Horizon and thermodynamic scales.** The Schwarzschild radius is $r_s = 2M$ and the isotropic locking radius is $\rho_h = M/2$ (Section 3); the surface gravity is $\kappa_{\text{sg}} = 1/(4M)$ and therefore
$$T_H = \frac{\kappa_{\text{sg}}}{2\pi} = \frac{1}{8\pi M}, \qquad S_{\text{BH}} = \frac{A}{4} = 4\pi M^2 \tag{53}$$
in geometric units (Section 4).

- **Evaporation scalings.** In the semiclassical regime, integrating the graybody-corrected energy flux yields a mass-loss law of the schematic form $dM/dt = -\mathcal{P}(M)$, with $\mathcal{P}(M) \propto M^{-2}$ when emission is dominated by effectively massless species. Consequently, the evaporation time scales as $t_{\text{evap}} \propto M_0^3$ for an initial mass M_0, and the Page-time turnover occurs at the same cubic order [37, 38, 51].

- **Scan-time calibration.** Under the modular/KMS organizing principle (Assumption 5.1), the 2π-normalized modular flow in the Rindler limit provides a quantitative way to fix the scan step δ by matching the KMS temperature to T_H (Remark after Assumption 5.1).

Limitations and Open Problems

Conjecture 6.1 (Mismatch-to-Poisson). *Fix a microscopic scan–projection readout model as in Paper I [31]. Let $\sigma_\varepsilon(\mathbf{x})$ be the coarse-grained mismatch density induced by the local orbit/discrepancy functionals (window-symmetric-difference flips and prefix-count deviations) at spatial resolution ε. Suppose σ_ε admits a continuum limit σ (in a distributional or weak sense) and that the phase-potential functional converges to the Dirichlet form. Then the continuum phase potential Φ is characterized by the Poisson equation with source σ (Appendix A.2) and reproduces the Newtonian sector. In particular, this supplies a microscopic derivation target for the mismatch-source input used in Assumption 2.7.*

Minimal verification criteria. This closure reduces to: (i) define σ_ε from explicit microscopic readout data (Paper I), (ii) prove a scaling/compactness statement ensuring $\sigma_\varepsilon \Rightarrow \sigma$, and (iii) identify the continuum limit of the discrete orbit-cost functional with the Dirichlet energy (Appendix A.2). Step (iii) fits the standard discrete-to-continuum paradigm for quadratic energies (e.g. via Γ-convergence) [8], while the passage from weak sources to Poisson solutions is controlled by classical elliptic PDE theory [14]. Quantitative discrepancy control supplies the needed uniform bounds on the microscopic readout deviations [10, 24, 27].

Conjecture 6.2 (Causal (4D) completion of inversion). *There exists a Lorentzian spacetime realization of Omega Dynamics whose exterior region is isometric to Schwarzschild, whose near-horizon geometry reduces to the Rindler$\times S^2$ form (Section 4), and whose continuation across the locking surface implements the inversion rule (Axiom 3.6) as a causal gluing to a boundary readout channel.*

Minimal verification criteria. A causal completion can be formulated as a concrete construction problem: specify a Lorentzian metric and a gluing map that (i) matches the exterior uniquely up to diffeomorphism [50], (ii) respects the classical Einstein–Rosen/Kruskal template in the time-symmetric limit [13, 26], and (iii) yields a well-posed exterior initial-value problem with effective boundary/readout degrees of freedom (compatible with the membrane/ER=EPR organizing pictures [33, 47]). At the level of Lorentzian geometry, such gluings are naturally formulated in the junction-condition framework for hypersurfaces, including the null (horizon) limit [1, 19, 40].

Conjecture 6.3 (Semiclassical Hawking matching from modular/KMS structure)**.** *Assume the modular-scan identification (Assumption 5.1) and the near-horizon Rindler form (Section 4). If the reduced state on the exterior observable algebra satisfies the corresponding KMS condition with respect to the boost/modular flow (as in the Bisognano–Wichmann paradigm for wedge algebras [4, 5, 15]), then the readout marginals exhibit Hawking–Unruh thermality at* $T_H = \kappa_{\rm sg}/(2\pi)$ *up to model-dependent graybody and backreaction corrections [3, 16, 17, 23, 37, 48, 50, 51].*

Minimal verification criteria. To match semiclassical observables one must: (i) compute detector response/two-point functions in the readout channel and verify KMS thermality at T_H, (ii) include the exterior potential-barrier transmission (graybody factors) to obtain the emitted spectrum [37], and (iii) control backreaction as a perturbation consistent with the unitary inversion/readout channel picture [50, 51].

A Auxiliary Derivations and Standard Results

A.1 Poisson Source Implies a 1/r Phase Potential

Let Φ be a static scalar potential on $\mathbb{R}^3$ solving the Poisson equation in the sense of distributions,

$$\Delta\Phi = 4\pi\mathcal{Q}\,\delta_0. \tag{54}$$

Then Φ is harmonic on $\mathbb{R}^3 \setminus \{0\}$ and, assuming spherical symmetry, satisfies for $r > 0$ the ODE

$$0 = \Delta\Phi(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\Phi'(r)\right). \tag{55}$$

Thus $r^2\Phi'(r) = C$ and $\Phi(r) = -C/r + \text{const}$. The constant is fixed by the flux normalization: integrating $\Delta\Phi = 4\pi\mathcal{Q}\delta_0$ over a ball B_R and applying the divergence theorem gives

$$4\pi\mathcal{Q} = \int_{B_R} \Delta\Phi\, dV = \int_{\partial B_R} \nabla\Phi \cdot d\mathbf{A} = 4\pi R^2\Phi'(R), \tag{56}$$

so $R^2\Phi'(R) = \mathcal{Q}$ and therefore

$$\Phi(r) = -\frac{\mathcal{Q}}{r} + \text{const}. \tag{57}$$

This is the standard three-dimensional $1/r$ kernel, with the overall sign fixed by the chosen Poisson convention [20].

A.2 Dirichlet Principle for the Static Phase Potential

Let ρ_Φ be a prescribed (compactly supported) source density on $\mathbb{R}^3$ and consider the functional

$$\mathcal{F}[\Phi] := \int_{\mathbb{R}^3} \left(\frac{1}{8\pi}|\nabla\Phi|^2 + \rho_\Phi\,\Phi\right) d^3x, \tag{58}$$

over sufficiently regular test functions with $\Phi(\mathbf{x}) \to 0$ as $|\mathbf{x}| \to \infty$. Varying $\Phi \mapsto \Phi + \epsilon\,\eta$ with compactly supported η and integrating by parts gives

$$\frac{d}{d\epsilon}\mathcal{F}[\Phi + \epsilon\eta]\Big|_{\epsilon=0} = \int_{\mathbb{R}^3} \left(-\frac{1}{4\pi}\Delta\Phi + \rho_\Phi\right) \eta\, d^3x, \tag{59}$$

so stationarity for all η implies the Poisson equation

$$\Delta\Phi = 4\pi\rho_\Phi. \tag{60}$$

This variational characterization is standard in potential theory; see, e.g., [20] for the electrostatic case (note that sign conventions for Poisson's equation may differ).

A.3 Poisson Equation Implies Gauss Law

Assume Φ is sufficiently regular on a bounded region $V \subset \mathbb{R}^3$ with smooth boundary ∂V and satisfies

$$\Delta\Phi = 4\pi\rho_\Phi \tag{61}$$

in the distributional sense on V. Integrating and applying the divergence theorem gives

$$\oint_{\partial V} \nabla\Phi \cdot d\mathbf{A} = \int_V \Delta\Phi \, d^3x = 4\pi \int_V \rho_\Phi \, d^3x. \tag{62}$$

Thus, defining the total phase charge in V by $\mathcal{Q}_\Phi(V) := \int_V \rho_\Phi \, d^3x$, one obtains the Gauss-law form

$$\oint_{\partial V} \nabla\Phi \cdot d\mathbf{A} = 4\pi\mathcal{Q}_\Phi(V), \tag{63}$$

which in particular yields $\oint \nabla\Phi \cdot d\mathbf{A} = 4\pi\mathcal{Q}$ for a point source $\rho_\Phi = \mathcal{Q}\delta_0$. See [20] for the standard potential-theoretic formulation.

A.4 Irrational Rotation Coding and Sturmian Sequences

Fix $\alpha \in (0,1) \setminus \mathbb{Q}$ and $\beta \in [0,1)$. Consider the Kronecker orbit $x_n = \{n\alpha + \beta\} \in [0,1)$ and the two-interval partition $[0, 1-\alpha) \cup [1-\alpha, 1)$. Its itinerary is the binary sequence

$$s_n := \lfloor (n+1)\alpha + \beta \rfloor - \lfloor n\alpha + \beta \rfloor \in \{0,1\}. \tag{64}$$

This s_n is a (lower) *mechanical word.* A classical theorem in symbolic dynamics states that for irrational α, mechanical words are precisely the (aperiodic) *Sturmian* sequences, characterized equivalently by minimal subword complexity $p(m) = m+1$ among all aperiodic bi-infinite binary sequences [28, 29, 35].

For coarse-grained observables, Weyl equidistribution implies the orbit $\{n\alpha + \beta\}$ is uniformly distributed mod 1 [52], so single-step statistics can appear thermal even when the underlying sequence is deterministic. The information capacity in Omega Dynamics is attributed to the long-range correlations of the itinerary (which are not visible to marginal statistics).

A.5 Modular Flow Is Unitary

In Tomita–Takesaki theory, given a von Neumann algebra $\mathcal{A}$ and cyclic separating vector $|\Omega\rangle$, the modular operator Δ_Ω is positive self-adjoint and Δ_Ω^{it} is unitary for every $t \in \mathbb{R}$, implementing the modular automorphism group $\sigma_t(A) = \Delta_\Omega^{it} A \Delta_\Omega^{-it}$ [46].

A.6 Newtonian Limit of the Einstein Equation

Assume a weak, static field in which the metric takes the standard Newtonian form [50]

$$ds^2 = -(1 + 2\phi)\, dt^2 + (1 - 2\phi)\, \delta_{ij}\, dx^i dx^j, \qquad |\phi| \ll 1, \tag{65}$$

with $\phi = \phi(\mathbf{x})$ and negligible pressures, so that $T_{00} \approx \rho_m$ and other components are subleading. Then one has, to leading order in ϕ,

$$G_{00} = 2\,\Delta\phi + O(\phi\,\partial^2\phi, (\partial\phi)^2), \tag{66}$$

so the field equation $G_{\mu\nu} = \kappa_G T_{\mu\nu}$ reduces to

$$\Delta\phi = \frac{\kappa_G}{2}\,\rho_m, \tag{67}$$

which fixes $\kappa_G = 8\pi$ by matching to $\Delta\phi = 4\pi\rho_m$ in geometric units [50].

A.7 Restoring Physical Units

Throughout the paper we set $c = G = \hbar = k_B = 1$ (geometric units), so the Schwarzschild mass parameter M has dimensions of length. Restoring SI units for a physical mass m gives

$$M = \frac{Gm}{c^2}, \qquad r_s = 2M = \frac{2Gm}{c^2}. \tag{68}$$

The Hawking temperature and Bekenstein–Hawking entropy take the standard forms

$$T_H = \frac{\hbar c^3}{8\pi G k_B\, m}, \qquad S_{\mathrm{BH}} = \frac{k_B c^3}{4G\hbar}\,A, \qquad A = 4\pi r_s^2, \tag{69}$$

which reduce to $T_H = 1/(8\pi M)$ and $S_{\mathrm{BH}} = A/4$ in geometric units [3, 16, 17, 51].

A.8 Scaling estimate for Hawking power and lifetime

Independently of the detailed graybody factors, the leading scaling of the Hawking power with mass follows from dimensional analysis. In geometric units, the horizon area scales as $A \sim M^2$ and the temperature scales as $T_H \sim 1/M$. A naive blackbody estimate gives

$$P \sim A T_H^4 \sim M^2 \left(\frac{1}{M}\right)^4 \sim \frac{1}{M^2}, \tag{70}$$

so that the mass-loss law takes the schematic form $dM/dt \sim -\alpha/M^2$ with a dimensionless coefficient α set by graybody transmission and particle content. Integrating yields the cubic lifetime scaling $t_{\mathrm{evap}} \sim M_0^3/(3\alpha)$ for an initial mass M_0. For quantitative coefficients and graybody-corrected spectra, see Page's classic computation for massless fields [37] and standard references [3, 51].

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