Holographic Phase Thermodynamics: Arithmetic Statistical Mechanics, Computational Lapse, and the Geometric Origin of Intelligence in HPA–Ω

Haobo Ma*
AELF PTE LTD.
#14-02, Marina Bay Financial Centre Tower 1, 8 Marina Blvd, Singapore 018981

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Abstract

In the HPA–Ω program, the universe is modeled ontically by a single normalized global state and an intrinsic unitary scan; "time," "probability," and macroscopic irreversibility are not postulated but induced by finite-resolution holographic readout [1–3]. This paper develops the statistical mechanics of that interface. We introduce *Arithmetic Statistical Mechanics* (ASM), in which microstates are deterministic phase orbits of a Weyl scan and macrostates are the finite-depth symbol statistics produced by a scan–projection readout protocol (O5). The central claim is that thermodynamic entropy is *phase friction*: the accumulated arithmetic mismatch generated when a continuous unitary orbit is projected onto a discrete distinguishable readout lattice. In the golden branch, this lattice is canonically organized by Zeckendorf/Fibonacci coding.

We define phase friction using one-dimensional star discrepancy and its accumulated mismatch $E_N = ND_N^*$ for irrational rotations. For badly approximable scan slopes, E_N admits an explicit logarithmic upper bound, while rational slopes exhibit linear growth corresponding to periodic "thermal death" by phase locking [4, 5]. We then reinterpret gravity as an entropic response: a coarse-grained mismatch density sources a phase potential Φ whose gradient is a phase pressure. Finally, by identifying gravitational redshift with a *computational lapse* field $\mathcal{N} = \kappa_0/\kappa$ determined by routing overhead [2, 6], we obtain a simple rescaling law for externally observed entropy production, $\mathrm{d}S/\mathrm{d}t \propto \sigma(x)\,\mathcal{N}(x)$, describing gravitational "computational cooling." We propose a physical definition of life and intelligence as an *active error-correction phase*: predictive feedback that pays a geometric Landauer cost to locally reduce phase friction and stabilize low-entropy structure.

We provide reproducible toy experiments (Python) validating the logarithmic mismatch bound for the golden branch, the periodicity of rational slopes, and the lapse rescaling of entropy flow.

Keywords: holographic readout; arithmetic statistical mechanics; star discrepancy; phase friction; Weyl pair; Zeckendorf coding; computational lapse; entropic gravity; geometric Landauer principle; intelligence as active error correction.

Conventions. Unless otherwise stated, log denotes the natural logarithm. "mod 1" refers to reduction in $\mathbb{R}/\mathbb{Z} \cong \mathbb{T}$. We set $c = 1$ in theoretical derivations unless explicitly restored. Throughout, "ontic" refers to the global unitary scan layer, while "operational" refers to finite-resolution readout and implementation constraints.

*Email: auric@aelf.io

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1 Introduction: from unitarity to irreversibility

Modern physics accommodates two time concepts that appear mutually incompatible:

- **Unitary time:** microscopic evolution is implemented by unitary operators and is, in principle, reversible.
- **Thermodynamic time:** macroscopic readout loses accessible information and exhibits irreversibility and entropy increase.

In the minimal axiomatization of the HPA–Ω framework [2, 3], this tension is not resolved by abandoning unitarity. Instead it is *layered*: the ontic layer is modeled by a single global state and intrinsic unitary scan; the operational layer is defined by finite-resolution scan–projection readout protocols constrained by holographic capacity [7–9]. "Probability" and "time" become properties of the readout interface, not primitive ingredients.

Thesis. This paper gives a micro-definitional, computable answer to "where does entropy come from?" in that setting:

Thermodynamic entropy is phase friction: arithmetic mismatch accumulated when a continuous unitary phase orbit is projected onto a discrete readout lattice.

In the golden branch, the readout lattice is canonically organized by Ostrowski/Zeckendorf coding; the mismatch admits sharp computable bounds, and periodic locking for rational slopes becomes a concrete model of "heat death by phase crystallization." These statements require no ad hoc stochastic noise assumption: the randomness is not ontic but readout-induced.

From thermodynamics to intelligence. Once entropy is understood as an interface cost, it becomes natural to interpret stable low-entropy structures as *error-correcting subsystems*. We propose that life and intelligence are not violations of the second law but its operational complement: an *active error-correction phase* that uses predictive feedback to locally suppress phase friction while exporting dissipation to an environment, paying a geometric Landauer cost.

1.1 Contributions and structure

- **Arithmetic Statistical Mechanics (ASM).** We define microstates as scan phase orbits and macrostates as finite-depth readout statistics and codes (Section 3).
- **Phase friction and entropy.** We define phase friction via star discrepancy and accumulated mismatch, derive computable upper bounds for badly approximable slopes, and interpret entropy increase as an operational coarse-graining monotonicity (Section 3).
- **First-law dictionary.** We propose a generalized first law in which energy is computational flux, temperature is scan tick rate, and work includes geometric routing overhead (Section 4).
- **Computational lapse and cooling.** We relate gravitational redshift to a computational lapse $\mathcal{N} = \kappa_0/\kappa$ and derive a rescaling law for externally observed entropy production (Section 5).
- **Gravity as phase pressure.** We interpret gravity as an entropic/phase-pressure response sourced by mismatch density, distinguishing it from purely statistical entropic gravity proposals [10] (Section 6).
- **Intelligence as active error correction.** We define agents operationally and propose a geometric Landauer principle with an impedance term (Section 8).

- **Reproducible toy experiments.** We provide minimal Python experiments validating mismatch growth bounds, rational lock-in, and lapse rescaling (Section 9 and Appendix E).

Outline. Section 2 summarizes the scan–projection readout interface used as the sole structural input. Sections 3–6 develop ASM, its thermodynamic dictionary, and the lapse rescaling principle. Sections 7–8 discuss the golden-branch third-law template and intelligence as a phase transition. Section 9 lists testable templates. Appendices record symbols, key formulas, and reproducible code.

2 Axioms and operational interface: scan–projection readout

This paper uses one structural input: the layered HPA–Ω interface separating (i) an ontic unitary scan from (ii) an operational finite-resolution readout. We state it in an intentionally minimal form, compatible with the fuller developments in [1–3].

2.1 Ontic layer: a static global state and intrinsic automorphism

Axiom 2.1 (O1: global state without external time)**.** *The physical universe is described by a normalized global state ω_Ω on an observable algebra $\mathcal{A}$. No externally imposed continuous time parameter is assumed.*

Axiom 2.2 (O2: finite information / holographic bound)**.** *For any causally closed region, the effective accessible Hilbert-space dimension obeys a holographic bound*

$$\dim \mathcal{H}_{\mathrm{region}} \;\leq\; \exp\left(\frac{A}{4\ell_P^2}\right), \tag{1}$$

where A is an area measure associated with the boundary of the region [7–9].

Axiom 2.3 (O3: intrinsic update)**.** *There exists a discrete-step automorphism $U : \mathcal{A} \to \mathcal{A}$ (implemented unitarily in controlled representations) such that correlations can be written as $\omega_\Omega(U^n(A))$ for $A \in \mathcal{A}$.*

Axiom 2.4 (O4: holographic encoding with approximate reconstruction)**.** *There exists an encoding map Φ relating bulk and boundary operator algebras such that the effective readout observables can be represented on a boundary algebra, supporting approximate reconstruction consistent with quantum error correction [11–13].*

2.2 Operational layer: readout as a finite-resolution instrument

Axiom 2.5 (O5: scan–projection readout and induced probability)**.** *Fix a resolution parameter $\varepsilon > 0$. A finite observer accesses the system through an operational* instrument $\{\mathcal{I}_k^{(\varepsilon)}\}_k$ *with associated POVM effects $\{E_k^{(\varepsilon)}\}_k \subset \mathcal{A}$ satisfying*

$$\sum_k E_k^{(\varepsilon)} = \mathbf{1}, \qquad E_k^{(\varepsilon)} \geq 0, \tag{2}$$

and the induced outcome probabilities

$$P_k^{(\varepsilon)} = \omega_{\mathrm{eff}}\left(E_k^{(\varepsilon)}\right), \tag{3}$$

where ω_{eff} denotes the effective state on the accessible readout sector.

Concretely, one may model readout by a "pointer" unitary V with spectral measure Π_V, and define effects using a window kernel (response function) $w_k^{(\varepsilon)}$ on $\mathbb{T} \cong \mathbb{R}/\mathbb{Z}$:

$$E_k^{(\varepsilon)} = \int_{\mathbb{T}} w_k^{(\varepsilon)}(x)\, \mathrm{d}\Pi_V(x). \tag{4}$$

Discarding outcomes yields the unconditional readout channel (CPTP map)

$$\Lambda^{(\varepsilon)}(\rho) = \sum_k \mathcal{I}_k^{(\varepsilon)}(\rho), \tag{5}$$

capturing operational irreversibility at fixed resolution [14, 15].

2.3 Weyl complementarity: the structural origin of readout noise

Axiom 2.6 (O6: Weyl pair and intrinsic non-commutativity)**.** *The scan is implemented by a unitary U_{scan}, and the pointer phase by a unitary V, satisfying a Weyl commutation relation*

$$U_{\mathrm{scan}} V = \mathrm{e}^{2\pi \mathrm{i} \alpha}\, V U_{\mathrm{scan}}, \qquad \alpha \in (0,1) \setminus \mathbb{Q}. \tag{6}$$

In the canonical representation on $L^2(\mathbb{T})$,

$$(U_{\mathrm{scan}}\psi)(x) = \psi(x+\alpha), \qquad (V\psi)(x) = \mathrm{e}^{2\pi \mathrm{i} x}\psi(x), \tag{7}$$

so the scan induces the irrational rotation orbit

$$x_n = x_0 + n\alpha \pmod 1. \tag{8}$$

The key point is operational: because U_{scan} and V fail to commute, one cannot refine "time access" and "phase readout" simultaneously without cost. In this paper, *thermal fluctuations and entropy production* are interpreted as the unavoidable residual mismatch generated when finite-resolution readout acts on this Weyl structure.

2.4 Golden branch and canonical coding

For a generic irrational slope α, windowed readout of the rotation orbit produces a Sturmian mechanical word and is canonically encoded by an Ostrowski numeration system [16–18]. The *golden branch* is the special choice

$$\alpha = \varphi^{-1} = \frac{\sqrt{5}-1}{2}, \qquad \varphi = \frac{1+\sqrt{5}}{2}, \tag{9}$$

whose continued fraction has constant partial quotients $[0; 1, 1, 1, \ldots]$. In this case, Ostrowski representation reduces to Zeckendorf/Fibonacci coding: every natural number admits a unique decomposition as a sum of non-consecutive Fibonacci numbers [19]. This provides a minimal-alphabet canonical discretization for readout depth and will serve as the normalization reference for "arithmetic" thermodynamic quantities in the next sections.

3 Arithmetic Statistical Mechanics: entropy as phase friction

Arithmetic Statistical Mechanics (ASM) is the statistical mechanics induced by scan–projection readout. It does not assume ontic randomness. Instead, it counts *readout equivalence classes* of a deterministic unitary orbit under finite resolution.

3.1 Microstates, macrostates, and the readout lattice

Definition 3.1 (Microstate and macrostate in ASM)**.** *Fix a local effective degree of freedom with pointer coordinate $x \in \mathbb{T}$. The **microstate** is the scan orbit $\{x_n\}_{n\geq 1}$ generated by O6. For a fixed readout resolution ε (O5), the **macrostate** is the discrete output sequence $\{k_n\}$ (or an induced symbolic word $\{s_n\}$) produced by windowing the orbit, together with a finite set of statistics (histograms, correlations, coding depth) computed from a length-N window.*

The finite readout resolution induces a discrete *readout lattice*: a partition of $\mathbb{T}$ into finitely many distinguishable cells. In the golden branch, the natural notion of lattice depth is the Zeckendorf coding depth associated with the Ostrowski system; heuristically, "spatial resolution" becomes "coding depth."

3.2 Phase friction as discrepancy

To quantify mismatch without adding stochasticity, we use star discrepancy for the Kronecker sequence $\{x_n\}$.

Definition 3.2 (Star discrepancy and accumulated mismatch)**.** *Let $x_1, \ldots, x_N \in [0,1)$ be the first N points of the scan orbit. The one-dimensional star discrepancy is*

$$D_N^* := \sup_{0\leq a\leq 1} \left| \frac{1}{N} \#\{1 \leq n \leq N : \ x_n < a\} - a \right|. \tag{10}$$

The accumulated mismatch (worst-case counting deviation) is

$$E_N := N D_N^*. \tag{11}$$

Theorem 3.3 (Koksma's inequality: discrepancy controls readout error)**.** *Let $x_1, \ldots, x_N \in [0,1)$ and let $f : [0,1) \to \mathbb{R}$ have bounded variation $V(f)$ in the sense of Hardy–Krause (in one dimension this reduces to total variation). Then*

$$\left| \frac{1}{N} \sum_{n=1}^{N} f(x_n) - \int_0^1 f(x)\,\mathrm{d}x \right| \leq V(f)\, D_N^*. \tag{12}$$

Theorem 3.3 provides an operational meaning of phase friction: D_N^* upper-bounds the worst-case readout bias for any bounded-variation observable at the readout layer. Standard references include [4, 20].
In ASM we interpret:

- D_N^* as **phase friction strength** (a per-sample readout mismatch intensity);
- E_N as **accumulated phase friction** (a finite-window "heat" generated by mismatch).

Theorem 3.4 (Logarithmic bound for badly approximable slopes)**.** *Let $x_n = x_0 + n\alpha \bmod 1$ with α irrational. If α has bounded continued-fraction coefficients (equivalently, α is badly approximable), then there exists a constant $C(\alpha) > 0$ such that*

$$D_N^* \leq C(\alpha) \frac{\log N}{N} \qquad \textit{for all } N \geq 2, \tag{13}$$

and hence

$$E_N \leq C(\alpha) \log N. \tag{14}$$

The golden branch $\alpha = \varphi^{-1}$ is extremal among badly approximable slopes in the Hurwitz sense, providing near-minimal constants in such bounds [4, 5].

An explicit continued-fraction bound controlling D_N^* by partial quotients is recorded in Appendix B, Eq. (55).

Proposition 3.5 (Periodic "thermal death" for rational slopes)**.** *If $\alpha = p/q \in \mathbb{Q}$ in lowest terms, then the orbit has period q and the readout is eventually periodic (with period dividing q) under any fixed finite-resolution window protocol. Moreover, for every N that is a multiple of q, one has the explicit star-discrepancy lower bound*

$$D_N^* \geq \frac{1}{2q}, \tag{15}$$

and therefore the accumulated mismatch grows at least linearly:

$$E_N = N D_N^* \geq \frac{N}{2q}. \tag{16}$$

A short proof of the universal $1/(2q)$ lower bound is given in Appendix B.
Proposition 3.5 expresses an operational form of heat death: phase locking collapses accessible macrostates into a short periodic cycle, and mismatch accumulates at a constant per-step rate under fixed finite resolution.

3.3 Entropy as arithmetic mismatch: two compatible definitions

ASM matches the HPA layer-separation principle: additive thermodynamic quantities arise from log readout of multiplicative weights. This yields two compatible entropies.

Definition 3.6 (Readout log-weight entropy)**.** *Let $w \in (0, 1]$ denote the operational success weight of a macro-constraint (e.g. a discrimination task) under a fixed readout protocol. Define*

$$S_{\mathrm{rd}} := k_B \, (-\log w). \tag{17}$$

Definition 3.7 (Phase-friction entropy)**.** *For a length-N window define the phase-friction entropy*

$$S_{\mathrm{pf}}(N) := k_B \, E_N = k_B \, N D_N^*. \tag{18}$$

S_{rd} and S_{pf} are calibrated differently but share the same structural meaning: entropy quantifies the operational cost of making a coarse-grained description consistent with a continuously scanned ontic orbit. In the golden branch, the cost can be normalized by Zeckendorf depth, making S_{pf} a directly computable proxy for thermodynamic entropy flow in toy models.

3.4 Second law as monotonicity under readout coarse-graining

Operational irreversibility is encoded by the unconditional readout channel $\Lambda^{(\varepsilon)}$ (O5), which forgets the outcome record. A key point is that for the POVM $\{E_k^{(\varepsilon)}\}$ induced by readout, there exists a canonical "square-root" (Lüders-type) instrument with Kraus operators $M_k = \sqrt{E_k^{(\varepsilon)}}$ [14, 15]. Its unconditional channel

$$\Lambda^{(\varepsilon)}(\rho) = \sum_k \sqrt{E_k^{(\varepsilon)}} \, \rho \, \sqrt{E_k^{(\varepsilon)}} \tag{19}$$

is automatically unital because $\Lambda^{(\varepsilon)}(\mathbb{1}) = \sum_k E_k^{(\varepsilon)} = \mathbb{1}$. Therefore, entropy monotonicity follows without an extra assumption.

Proposition 3.8 (Arithmetic second law (interface form))**.** *Let ρ be an effective state on the accessible sector, and let $\Lambda^{(\varepsilon)}$ be the outcome-discarding readout channel associated with the square-root instrument of the induced POVM (O5). Then $\Lambda^{(\varepsilon)}$ is unital and the von Neumann entropy satisfies*

$$S\Big(\Lambda^{(\varepsilon)}(\rho)\Big) \geq S(\rho). \tag{20}$$

Moreover, the phase-friction entropy S_{pf} provides a computable upper proxy for the coarse-grained entropy production in rotation models, controlled by discrepancy bounds such as Theorem 3.4.

Proof. Let d be the Hilbert-space dimension of the effective sector and let $\tau := \mathbb{1}/d$ be the maximally mixed state. For any CPTP map Λ, quantum relative entropy satisfies the data-processing inequality

$$D(\rho\|\tau) \;\geq\; D(\Lambda(\rho)\|\Lambda(\tau)), \tag{21}$$

see e.g. [15]. If Λ is unital then $\Lambda(\tau) = \tau$. Using $D(\rho\|\tau) = \log d - S(\rho)$, one obtains

$$\log d - S(\rho) \;\geq\; \log d - S(\Lambda(\rho)), \tag{22}$$

equivalently $S(\Lambda(\rho)) \geq S(\rho)$. □

Remark 3.9 (Why no external noise is needed)**.** *Theorem 3.4 is purely arithmetic: even a perfectly deterministic unitary scan produces mismatch at finite readout resolution. In this sense, "thermal noise" is structural—it is the residual of projecting a Weyl orbit onto a discrete readout lattice.*

4 Generalized first law: energy as computational flux

To elevate phase friction from a mismatch measure to a thermodynamic quantity, we need a dictionary relating entropy to energy, temperature, and work. In the scan–projection semantics, energy is not a primitive "substance"; it is the operational *computational flux* required to (i) perform finite-resolution readout, and (ii) maintain and repair structure against phase friction.

4.1 Computational temperature as tick rate

Let τ denote scan-time (intrinsic step count) and t denote external/operational time. Define the local scan tick rate

$$\nu(x) := \frac{\mathrm{d}\tau}{\mathrm{d}t}. \tag{23}$$

In the Ω implementation dictionary, ν is controlled by routing overhead through a lapse field $\mathcal{N}$ (Section 5). We define the *computational temperature* by a proportionality

$$T_c(x) := \eta_T\, \nu(x), \tag{24}$$

where η_T sets units (one may take $\eta_T = 1$ in natural units). The operational meaning is direct: faster local ticking forces more scan samples through the same finite-resolution readout channel per unit t, increasing mismatch generation rate.

Quantum speed-limit calibration. In microscopic quantum implementations, the tick rate of distinguishable state updates is bounded by energy-time speed limits. For example, the Margolus–Levitin bound implies that a system with average energy above its ground state satisfies

$$\nu \;\leq\; \frac{2(\langle H\rangle - E_0)}{\pi\hbar}, \tag{25}$$

while the Mandelstam–Tamm bound implies $\nu \le 2\Delta H/(\pi\hbar)$ [21, 22]. Lloyd's "ultimate limits" estimate uses these bounds to relate energy to an upper bound on logical operations per unit time [23]. Therefore η_T can be fixed by calibration to such microscopic limits (e.g. by saturating an appropriate speed bound), or treated as an effective proportionality constant when the underlying microscopic degrees of freedom are not specified.

4.2 Structure information: Zeckendorf-effective bits

In the golden branch, readout depth is canonically organized by Zeckendorf/Fibonacci coding [19]. Let I denote the number of *effective* internal bits (or more generally, effective Ostrowski depth) that can be stably maintained under a given readout-and-control protocol. I measures *structure capacity*:

- larger I corresponds to finer internal models and more stable correlations;
- smaller I corresponds to coarse macrostates near equilibrium.

4.3 A first-law form for ASM

Proposition 4.1 (Generalized first law (computational flux form))**.** *At fixed readout protocol class (fixed ε and window family), the externally observed change of computational flux E decomposes as*

$$\mathrm{d}E = T_c\,\mathrm{d}S_{\mathrm{pf}} + \Xi\,\mathrm{d}I + \delta W_{\mathrm{geom}}. \tag{26}$$

Here S_{pf} is phase-friction entropy (Section 3), Ξ is the "chemical potential" conjugate to stable structure information I, and δW_{geom} is geometric work associated with changes in routing overhead / impedance.

Proposition 4.1 can be read as a bookkeeping identity in the standard thermodynamic sense [24]: for a fixed protocol class, one may treat E as an effective state function $E = E(S_{\mathrm{pf}}, I, \mathrm{geom})$ and *define* the conjugate variables by partial derivatives,

$$T_c := \left(\frac{\partial E}{\partial S_{\mathrm{pf}}}\right)_{I,\mathrm{geom}}, \qquad \Xi := \left(\frac{\partial E}{\partial I}\right)_{S_{\mathrm{pf}},\mathrm{geom}},$$

with δW_{geom} collecting protocol-dependent geometric work terms. The physical content is that finite-resolution readout forces an unavoidable tradeoff between dissipated mismatch and maintainable internal structure.

4.4 Landauer cost and log-cost additivity

The log structure of $S_{\mathrm{rd}} = k_B(-\log w)$ (Section 3) matches the multiplicative-to-additive conversion intrinsic to readout. In particular, when independent constraints have success weights w_1, w_2, their joint weight is $w_1 w_2$ and the total log-cost is additive.

Landauer's principle provides a lower bound on the energetic cost of erasing one bit at temperature T [25, 26]:

$$W_{\mathrm{erase}} \ge k_B T \ln 2. \tag{27}$$

In HPA–Ω, the relevant temperature is the computational temperature T_c and the total cost includes geometric impedance (Appendix D). This makes explicit how maintaining low-entropy structure is compatible with the second law: entropy is exported to the environment through an operationally irreversible readout-and-erase channel [27, 28].

5 Relativistic thermodynamics: computational lapse and gravitational cooling

This section connects entropy production to gravitational redshift via the Ω implementation dictionary: time dilation is computational slowdown.

5.1 Routing overhead and computational lapse

In the Ω program, microscopic evolution is modeled by a local unitary update (e.g. a PQCA step) that can be exactly compiled into a one-dimensional nearest-neighbor circuit on a finite region [2, 6]. The required circuit depth defines the *routing overhead* field $\kappa(x)$: the operational cost of moving information through local connectivity to realize the update. A reference region defines κ_0 and the *computational lapse*

$$\mathcal{N}(x) = \frac{\kappa_0}{\kappa(x)}, \qquad \mathrm{d}\tau_{\mathrm{loc}}(x) = \mathcal{N}(x)\,\mathrm{d}t. \tag{28}$$

Higher routing overhead means fewer realizable logical updates per unit background depth $\mathrm{d}t$, hence a slower local clock. Identifying $\mathcal{N}$ with the GR lapse gives an operational dictionary for redshift in static spacetimes [29–31].

5.2 Entropy-production rescaling

Let $S_{\mathrm{pf}}(x;\tau)$ denote the accumulated phase-friction entropy generated by a fixed readout protocol up to intrinsic scan time τ at location x (Section 3). We define the (possibly time-dependent) mismatch density by the scan-time derivative

$$\sigma(x,\tau) := \frac{1}{k_B}\frac{\mathrm{d}S_{\mathrm{pf}}(x;\tau)}{\mathrm{d}\tau}, \tag{29}$$

interpreted in discrete scan time as a difference quotient when needed. With this definition, the scan-time entropy production law is simply

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}\tau} = k_B\,\sigma(x,\tau). \tag{30}$$

Then the entropy production rate with respect to external time t is rescaled by the computational lapse:

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}t} = \frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} = k_B\,\sigma(x,\tau)\,\mathcal{N}(x). \tag{31}$$

Equation (31) is the thermodynamic content of gravitational time dilation in this framework:

- larger mismatch density σ produces more entropy per scan step;
- stronger gravity corresponds to smaller $\mathcal{N}$ (larger routing overhead), so external observers see *slower* entropy flow.

We call this effect **computational cooling**: in regions where computation is slowed by geometric overhead, the externally observed entropy-production rate is suppressed.

5.3 Interpretation and limits

The lapse rescaling does not imply that mismatch disappears in strong gravity; rather, it means that mismatch accumulation is “time-dilated” in the external description. When $\mathcal{N} \to 0$ (an idealized horizon limit in the GR dictionary), external entropy flow tends to zero, consistent with clock freezing. In Section 6 we connect the same mismatch density to a phase potential and a phase-pressure acceleration.

6 Gravity as entropic response: phase potential and phase pressure

This section summarizes the thermodynamic interpretation of the "phase pressure" mechanism developed in the HPA–Ω literature [6, 32]. The guiding idea is that coarse-grained readout mismatch acts as a source for an effective potential whose gradient generates an acceleration. In contrast to purely statistical entropic-gravity proposals [10], the source is *deterministic mismatch* induced by scan–projection incompatibility.

6.1 Mismatch density and phase potential

Let σ denote a coarse-grained mismatch density (Section 5). On macroscopic scales we define the phase potential Φ as the stationary point of a quadratic functional $S_\Phi[\Phi;\sigma]$ (Appendix C); equivalently, Φ satisfies the Poisson equation [33]:

$$\Delta\Phi = 4\pi\,\rho_\Phi, \qquad \rho_\Phi := \kappa_\Phi\,\sigma, \tag{32}$$

where κ_Φ is a coupling constant. Its normalization can be fixed by matching the exterior monopole term to the Newtonian potential, $\kappa_\Phi Q_\sigma = GM_g$ (Appendix C, Eq. (77)). The associated phase-pressure field is the conservative vector field

$$\mathbf{P}_\Phi := -\nabla\Phi. \tag{33}$$

For isolated localized sources, Eq. (32) yields a $1/r$ potential in the weak-field regime, recovering the Newtonian template at leading order (Appendix C).

6.2 Entropic-force form

Define a computational free energy functional

$$F_{\text{free}} := E - T_c S_{\text{pf}}, \tag{34}$$

where E is computational flux and T_c is computational temperature (Section 4). In a quasi-static approximation, effective dynamics tends to decrease F_{free}. For a degree of freedom with coordinate $\mathbf{x}$, the corresponding force is

$$\mathbf{F}_{\text{entropic}} := -\nabla_{\mathbf{x}} F_{\text{free}} \;=\; -\nabla_{\mathbf{x}} E + S_{\text{pf}}\nabla_{\mathbf{x}} T_c + T_c\,\nabla_{\mathbf{x}} S_{\text{pf}}. \tag{35}$$

In the isothermal and weak-backreaction regime ($\nabla T_c \approx 0$ and $|\nabla E|$ subleading for the effective degree of freedom), Eq. (35) reduces to the standard entropic-force form $\mathbf{F} \approx T_c\nabla S$ [24]. When S_{pf} is controlled by the mismatch density σ and σ sources Φ, the phase-pressure field $\mathbf{P}_\Phi$ provides a geometric representative of this entropic response. In words: matter flows toward configurations that reduce mismatch production and routing impedance.

6.3 Black holes as computational blackbodies

Black-hole thermodynamics suggests that horizons act like thermal boundaries [34–36]. In the present semantics, the horizon limit corresponds to a strong-overhead region where $\mathcal{N} \to 0$ (Section 5). External observers then see effective freezing of internal scan dynamics; mismatch that cannot be actively repaired by the slowed local computation is exported through the boundary readout channel and appears approximately thermal at the level of marginal statistics, while correlations can carry information [32].

This coexistence of thermal *marginals* with global unitarity is consistent with the modern understanding of the information paradox. In unitary evaporation, the fine-grained entropy of

Hawking radiation follows a Page curve [37]. In semiclassical gravity, the island formula (and related replica-wormhole methods) reproduces the Page curve and resolves the paradox within controlled settings [38, 39]. In HPT language: outcome-discarding readout channels naturally generate approximately thermal coarse-grained statistics, while the ontic unitary scan retains the correlations required for information recovery.

7 Third-law template and "heat death": the golden attractor

ASM makes the "heat death" problem concrete: if scan slopes are rational (or effectively rational at the accessible resolution), the orbit locks into a short cycle, collapsing the accessible macrostate space. Conversely, badly approximable slopes resist such locking.

7.1 Rational locking as periodic thermal death

For $\alpha = p/q$, the orbit visits only q phase points. Under any fixed finite-resolution window family, the induced readout becomes eventually periodic with period dividing q. The accumulated mismatch grows linearly (Proposition 3.5), so a fixed fraction of readout steps become irreparably "miscounted" with respect to the uniform reference. Operationally, this is a form of crystalline equilibrium: the system loses macroscopic openness by phase locking.

7.2 Why the golden branch is special

The mechanism preventing rapid periodic lock-in is arithmetic. Irrational slopes are approximated by rationals via continued fractions; small denominators yield short pseudo-periods. A slope is *badly approximable* if it cannot be too well approximated by rationals, equivalently if its continued-fraction coefficients are bounded [5]. Such slopes yield strong uniform-distribution control and hence logarithmic mismatch bounds (Theorem 3.4).

The golden branch $\alpha = \varphi^{-1} = [0; 1, 1, 1, \dots]$ is extremal in the classical Hurwitz/Markov sense. Hurwitz's theorem states that for any irrational α there exist infinitely many rationals p/q such that

$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{\sqrt{5}\, q^2}, \tag{36}$$

and the constant $\sqrt{5}$ is optimal: for $\alpha = \varphi^{-1}$ one has the *uniform* lower bound

$$\left|\varphi^{-1} - \frac{p}{q}\right| \geq \frac{1}{\sqrt{5}\, q^2} \qquad \text{for all } \frac{p}{q} \in \mathbb{Q}, \tag{37}$$

so φ^{-1} is among the hardest irrationals to approximate by rationals [5]. Operationally, this delays the onset of short pseudo-periods at a given approximation tolerance, maximizing resistance to phase-locking.

In ASM this means:

- long avoidance of short pseudo-periods (maximal resistance to phase locking),
- simultaneously, nonzero but controlled mismatch (entropy production remains finite and computable).

This motivates a third-law template: perfect "zero-friction" readout (vanishing mismatch at all depths) is unattainable for finite observers; attempting to suppress mismatch indefinitely drives the system toward periodic lock-in (loss of openness), rather than toward a frictionless continuum.

7.3 A structural route to $1/f$-type noise

Many complex systems exhibit $1/f$-type spectra, often associated with hierarchical time scales and near-criticality [40, 41]. In ASM the Zeckendorf/Fibonacci hierarchy provides a canonical ladder of time scales. When a readout protocol aggregates approximately equal-weight relaxation contributions across this ladder, $1/f$ behavior follows quantitatively (Appendix G). In the minimal closure where readout aggregates approximately equal-weight relaxation contributions across a Fibonacci (asymptotically geometric) ladder of characteristic times, one obtains a quantitative $1/f$ band with an explicit prefactor (Appendix G, Proposition G.1):

$$S(f) \;\approx\; \frac{w}{4f\,\log\varphi} \tag{38}$$

over the intermediate frequency range separating the smallest and largest Zeckendorf-resolved scales. More generally, bounded nonuniform ladder weights still yield a robust $1/f$ mid-band; the weights affect only the prefactor, which is bracketed between $w_-/(4f\log\varphi)$ and $w_+/(4f\log\varphi)$ (Appendix G, Corollary G.2).

8 The physical origin of intelligence: active error correction and reverse compilation

In a universe where phase friction is unavoidable for finite observers, stable low-entropy structure requires continuous repair. This section proposes an operational definition of life and intelligence as a phase transition: the emergence of predictive feedback loops that actively suppress local mismatch production while exporting dissipation.

8.1 Agents as predictive-feedback subsystems

Definition 8.1 (Agent)**.** *A subsystem $\mathcal{S}$ is an* agent *if, under finite computational flux E, it can:*

1. *acquire conditional information about its environment through a finite-resolution readout instrument (O5),*
2. *maintain an internal model that predicts future readout outcomes of the scan,*
3. *implement feedback operations that reduce its own phase-friction entropy production rate relative to the passive baseline.*

This definition converts Maxwell-demon language into engineering constraints: the demon does not violate the second law because it must pay readout and erasure costs [25–27]. What an agent accomplishes is not entropy elimination but entropy *redirection*: moving dissipation into an external waste channel while protecting internal structure.

8.2 Reverse compilation in the Ω dictionary

In the Ω implementation picture, local update rules are exactly compilable into nearest-neighbor circuits; the required depth is the routing overhead $\kappa(x)$ (Section 5). "Forward compilation" maps dynamics to required overhead; "reverse compilation" is the agentic operation: using limited internal bits I to infer which future phase information will be read out and to preconfigure control operations that cancel mismatch.

In HPA language, prediction corresponds to compressing future readout words in the canonical Zeckendorf/Ostrowski coordinate system, while control corresponds to applying phase corrections compatible with the Weyl structure. The net effect is a local reduction of mismatch density $\sigma(x)$ and therefore a reduction in S_{pf} production.

8.3 Geometric Landauer principle

Landauer's bound provides the minimal energetic cost of erasing one bit at temperature T [25]. In HPA–Ω, the operational temperature is T_c and the erasure must be implemented through a geometric communication network with routing overhead. This motivates a refined principle:

$$W_{\text{erase}} \geq k_B T_c \ln 2 + Z_{\text{geom}}, \tag{39}$$

where Z_{geom} is a geometric impedance term capturing the extra work required to route and re-encode information through constrained locality. Appendix D records a minimal formulation consistent with the additivity of log-costs.

8.4 Information-thermodynamic bound on predictive gain

Feedback control admits a quantitative upper bound: the work (or free-energy) advantage achievable by measurement-and-feedback is limited by the information acquired. In stochastic thermodynamics, generalized second-law inequalities take the form

$$\langle W \rangle \geq \Delta F - k_B T\, I, \tag{40}$$

where I is an appropriate mutual-information term between measurement outcomes and system degrees of freedom [42, 43]. Interpreting T as the computational temperature T_c in HPA–Ω, we obtain an operational bound on predictive free-energy gain:

$$\dot{F}_{\text{pred}} \;\leq\; k_B T_c\, \dot{I}_{\text{pred}}, \tag{41}$$

where $\dot{I}_{\text{pred}}$ denotes the mutual-information rate between the agent's internal state and future readout outcomes (a "thermodynamics of prediction" interface) [44].

8.5 Survival criterion and predictive efficiency

Let $\dot{F}_{\text{pred}}$ denote the rate of free-energy gain achieved by prediction-and-control, and let $\dot{W}_{\text{diss}}$ denote the dissipation rate required to maintain structure against phase friction. A necessary condition for sustained existence is

$$\dot{F}_{\text{pred}} > \dot{W}_{\text{diss}}. \tag{42}$$

Combining Eq. (42) with the information bound (41) yields a directly testable necessary condition in terms of an information rate:

$$\dot{I}_{\text{pred}} > \frac{\dot{W}_{\text{diss}}}{k_B T_c}. \tag{43}$$

Define the predictive efficiency

$$\eta_{\text{pred}} := \frac{\dot{F}_{\text{pred}}}{\dot{W}_{\text{diss}}}. \tag{44}$$

On this view, evolution is the search for architectures that maximize η_{pred} under constraints imposed by readout resolution, routing overhead, and mismatch density. Intelligence is then a phase transition in which an error-correcting feedback loop becomes self-sustaining and scalable in Zeckendorf depth.

9 Testable templates and reproducible toy experiments

The HPA–Ω program is a closed inference chain from scan–projection axioms to macroscopic templates. The appropriate empirical posture is to specify testable *interfaces*: spectral templates, scaling laws, and reproducible toy-model checks that can be extended in concrete microscopic implementations.

9.1 Holographic noise as a hierarchical spectral template

If an experimental setup is sensitive enough that instrumental thermal noise is subdominant, then the dominant residual may originate from finite-resolution readout mismatch rather than stochastic bath noise. ASM predicts that such mismatch need not be white: multi-scale coding (Ostrowski/Zeckendorf hierarchies) imprints a canonical ladder of time scales. In the minimal closure where readout aggregates approximately equal-weight relaxation contributions across the (asymptotically geometric) Fibonacci ladder, one obtains a quantitative $1/f$ mid-band with explicit prefactor (Appendix G, Proposition G.1 and Table 3):

$$S(f) \;\approx\; \frac{w}{4f\,\log\varphi}. \tag{45}$$

More generally, bounded nonuniform ladder weights still produce a robust $1/f$ mid-band; the weights affect only the prefactor (Appendix G, Corollary G.2). This provides a falsifiable interface: the slope, band limits, and prefactor are determined once the effective ladder range and the aggregation weight w are fixed by the instrument kernel and the scale flow.

9.2 Golden ratios in biological rhythms (as a control-law hypothesis)

ASM does not attribute mystical significance to the golden ratio. Instead, it yields a quantitative *anti-locking* criterion rooted in Diophantine approximation. In coupled-oscillator systems, phase locking occurs near rational resonances and is organized by Arnold tongues; high-order resonances (large denominators) are typically much narrower and therefore harder to lock into at fixed noise/coupling strength [45, 46].

Fix an operational locking tolerance $\delta > 0$ (set by noise level, coupling strength, or measurement resolution). Define the *resonance susceptibility index*

$$Q_\delta(\alpha) := \min\left\{q \in \mathbb{N} :\ \exists\, p \in \mathbb{Z} \text{ s.t. } \left|\alpha - \frac{p}{q}\right| < \delta\right\}. \tag{46}$$

If $Q_\delta(\alpha)$ is large, the system must access high-order resonances before it can phase-lock within tolerance δ.

Proposition 9.1 (Diophantine lower bound on resonance susceptibility)**.** *If α is badly approximable, then there exists a constant $c(\alpha) > 0$ such that*

$$\left|\alpha - \frac{p}{q}\right| \geq \frac{c(\alpha)}{q^2} \qquad \textit{for all } \frac{p}{q} \in \mathbb{Q} \tag{47}$$

see e.g. [5]. For any tolerance $\delta > 0$, this implies the quantitative anti-locking bound

$$Q_\delta(\alpha) \;\geq\; \left\lceil \left(\frac{c(\alpha)}{\delta}\right)^{1/2} \right\rceil. \tag{48}$$

By Hurwitz/Markov extremality (Section 7), the golden branch $\alpha = \varphi^{-1}$ achieves the maximal uniform constant $c(\varphi^{-1}) = 1/\sqrt{5}$ and satisfies the sharp lower bound

$$\left|\varphi^{-1} - \frac{p}{q}\right| \geq \frac{1}{\sqrt{5}\,q^2} \quad\Rightarrow\quad Q_\delta(\varphi^{-1}) \;\geq\; \left\lceil \left(\frac{1}{\sqrt{5}\,\delta}\right)^{1/2} \right\rceil, \tag{49}$$

which is (in the Hurwitz sense) maximal among irrational ratios [5]. This motivates a concrete control-law hypothesis: if an adaptive oscillator network is selected to resist low-order phase locking at a given effective tolerance δ, it should favor badly-approximable ratios, with the golden branch as the extremal candidate. A natural statistical test is to estimate δ from observed coupling/noise and compare the empirical distribution of inferred ratios against the predicted anti-locking index Q_δ.

9.3 Cosmological entropy flow and dark-energy parameterization

If dark energy is reinterpreted as a background "openness" budget required to prevent global periodic locking, then it is natural to associate an effective energy density $\rho_\sigma(a)$ to a cosmological mismatch reservoir. In an FLRW background, covariant energy conservation implies the continuity equation

$$\frac{\mathrm{d}\log\rho_\sigma}{\mathrm{d}\log a} = -3(1 + w_\sigma(a)), \tag{50}$$

so the equation-of-state parameter is determined by the scale flow:

$$w_\sigma(a) = -1 - \frac{1}{3}\frac{\mathrm{d}\log\rho_\sigma}{\mathrm{d}\log a}. \tag{51}$$

In particular, a stationary background mismatch reservoir ($\mathrm{d}\rho_\sigma/\mathrm{d}a \approx 0$ on Hubble time scales) predicts $w_\sigma \approx -1$ [47, 48]. Deviations from -1 directly measure the slow running of the mismatch reservoir and therefore provide a clean observational interface for the "openness budget" interpretation.

9.4 Reproducible toy experiments

Appendix E provides minimal Python scripts validating:

- logarithmic mismatch-growth compatibility for the golden branch (Theorem 3.4),
- linear mismatch growth and periodicity for rational slopes (Proposition 3.5),
- lapse rescaling of externally observed entropy flow (Eq. (31)),
- least-squares envelope fits for mismatch templates (Appendix F),
- a toy $1/f$ spectrum from Fibonacci/geometric relaxation ladders (Appendix G).

These experiments are not intended as definitive phenomenology; they are unit tests of the interface claims in the simplest controlled setting.

9.5 Quantitative parameter identification and fits (toy model)

The mismatch-growth bounds of Section 3 can be turned into measurable parameters. For badly approximable slopes, discrepancy theory implies the asymptotic template

$$E_N^{\uparrow}(\alpha; x_0) = C_\alpha(x_0)\,\log N + B_\alpha(x_0) + o(1), \tag{52}$$

while rational slopes satisfy a linear lower bound $E_N \geq N/(2q)$ for $\alpha = p/q$ (Proposition 3.5). In the simplest periodic case $\alpha = 1/2$, E_N is exactly linear with an explicit coefficient depending on the initial phase x_0 (Appendix B).

We perform least-squares fits of these templates to the toy data generated by the scripts in Appendix E. The fitted constants (including goodness-of-fit metrics) are reported in Appendix F, providing a concrete calibration of "phase friction" in the minimal 1D model.

10 Conclusion

Holographic Phase Thermodynamics (HPT) provides a unified operational picture in the HPA–Ω framework:

- Ontically, the universe is unitary and information-preserving.

- Operationally, finite-resolution scan–projection readout induces probability, thermodynamic time, and irreversibility.
- Entropy admits a micro-definitional origin as phase friction: accumulated arithmetic mismatch of a Weyl scan orbit under discrete readout.
- Gravitational redshift is computational slowdown governed by a lapse $\mathcal{N} = \kappa_0/\kappa$, implying computational cooling of externally observed entropy flow.
- Gravity can be modeled as an entropic/phase-pressure response sourced by mismatch density.
- Life and intelligence are active error-correction phases: predictive feedback that locally suppresses phase friction while paying a geometric Landauer cost.

The central shift is semantic but testable: instead of treating thermodynamics as an emergent average over unknown microstates, ASM treats it as an inevitable interface cost of finite readout acting on a deterministic unitary scan. The resulting scaling laws and spectral templates invite targeted tests once explicit microscopic realizations and measurement kernels are fixed.

A Symbols and notation

This appendix records frequently used symbols.

- ω_Ω: normalized global state on the observable algebra $\mathcal{A}$.
- $\mathcal{A}$: (effective) observable algebra; $A \in \mathcal{A}$ denotes an observable.
- $U : \mathcal{A} \to \mathcal{A}$: intrinsic discrete-step automorphism (update).
- U_{scan}: unitary scan operator in the Weyl pair (O6).
- V: pointer unitary; Π_V: its spectral measure.
- $\alpha \in (0,1) \setminus \mathbb{Q}$: scan slope in the Weyl commutation relation.
- $\mathbb{T} \cong \mathbb{R}/\mathbb{Z}$: circle (phase space); "mod 1" denotes reduction in $\mathbb{T}$.
- $\varepsilon > 0$: readout resolution parameter.
- $\{\mathcal{I}_k^{(\varepsilon)}\}$: readout instrument; $\{E_k^{(\varepsilon)}\}$: associated POVM effects; $\Lambda^{(\varepsilon)} = \sum_k \mathcal{I}_k^{(\varepsilon)}$: outcome-discarding channel.
- $x_n = x_0 + n\alpha$ mod 1: scan orbit points on $\mathbb{T}$.
- D_N^*: one-dimensional star discrepancy of $\{x_1, \ldots, x_N\}$.
- $E_N = ND_N^*$: accumulated mismatch.
- $S_{\text{rd}} = k_B(-\log w)$: log-weight readout entropy proxy.
- $S_{\text{pf}}(N) = k_B E_N$: phase-friction entropy for a length-N window.
- $\sigma(x)$: mismatch density (entropy production per scan step) at location x.
- κ_Φ: coupling constant in the phase-potential source $\rho_\Phi = \kappa_\Phi \sigma$.
- $Q_\sigma = \int_{\mathbb{R}^3} \sigma(\mathbf{x})\, \mathrm{d}^3\mathbf{x}$: total mismatch charge of a localized source.

- $M_\Phi = \kappa_\Phi Q_\sigma$: exterior monopole strength in $\Phi(r) \sim -M_\Phi/r$.
- $\kappa(x)$: routing overhead (compilation depth) at x; κ_0 reference overhead.
- $\mathcal{N}(x) = \kappa_0/\kappa(x)$: computational lapse (slowdown factor).
- τ: intrinsic scan time (step count); t: external/operational time.
- $\nu = \mathrm{d}\tau/\mathrm{d}t$: local tick rate; $T_c = \eta_T \nu$: computational temperature.
- E: computational flux (available resource rate); I: effective stable structure bits; Ξ: conjugate chemical potential.
- Φ: phase potential; $\mathbf{P}_\Phi = -\nabla\Phi$: phase pressure.
- Z_{geom}: geometric impedance term in the geometric Landauer principle.
- ζ_{geom}: dimensionless constant in $Z_{\text{geom}} = \zeta_{\text{geom}}\, k_B T_c \log(\kappa/\kappa_0)$.

B Discrepancy bounds and entropy proxies

This appendix records explicit arithmetic bounds for the Kronecker sequence $x_n = x_0 + n\alpha \bmod 1$ and their interpretation as entropy proxies in ASM.

B.1 Definitions

Given points $x_1, \dots, x_N \in [0,1)$, define the empirical distribution function

$$F_N(a) := \frac{1}{N}\#\{1 \le n \le N :\ x_n < a\}, \qquad 0 \le a \le 1. \tag{53}$$

The star discrepancy and accumulated mismatch are

$$D_N^* := \sup_{0 \le a \le 1} |F_N(a) - a|\,, \qquad E_N := N D_N^*. \tag{54}$$

B.2 Continued fractions and an explicit bound

Let $\alpha = [0; a_1, a_2, \dots]$ be the continued fraction of α, with convergents p_m/q_m. For each $N \ge 1$, choose m such that $q_m \le N < q_{m+1}$. Classical results bound the discrepancy of the Kronecker sequence in terms of the partial quotients [4, 5]:

$$D_N^* \;\le\; \frac{1}{N}\left(1 + \sum_{i=1}^{m} a_i\right). \tag{55}$$

A proof-level bound (Denjoy–Koksma + Ostrowski). For completeness we record a standard proof route (with explicit constants) based on the Denjoy–Koksma inequality for rotations and Ostrowski block decomposition.

Theorem B.1 (Ostrowski–Denjoy–Koksma bound for interval counts)**.** *Let $x_n = x_0 + n\alpha \bmod 1$ with α irrational. Let $W \subset [0,1)$ be any interval and define*

$$S_N(W) := \sum_{n=0}^{N-1} \mathbf{1}_W(x_0 + n\alpha). \tag{56}$$

Write the Ostrowski expansion $N = \sum_{k=0}^{m} b_k q_k$ (relative to α). Then for every $x_0 \in [0,1)$,

$$|S_N(W) - N\,\mu(W)| \le 2\sum_{k=0}^{m} b_k \le 2\sum_{k=0}^{m} a_{k+1}, \tag{57}$$

where μ is Lebesgue measure.

Proof. Let $f := \mathbf{1}_W - \mu(W)$. Then f has bounded variation $\mathrm{Var}(f) = 2$ and $\int_0^1 f \, \mathrm{d}\mu = 0$. For each $k \geq 0$, the Denjoy–Koksma inequality for rotations gives a uniform bound at convergent times:

$$\left| \sum_{j=0}^{q_k - 1} f(x_0 + j\alpha) \right| \leq \mathrm{Var}(f) = 2, \tag{58}$$

see e.g. [20,49,50]. Now expand $N = \sum_{k=0}^{m} b_k q_k$ and decompose the length-N sum into b_m blocks of length q_m, then b_{m-1} blocks of length q_{m-1}, and so on (Ostrowski block decomposition; see e.g. [17, 18]). Applying the q_k-time bound to each block yields the first inequality. The second follows from the standard digit bounds $b_0 < a_1$ and $b_k \leq a_{k+1}$. □

Corollary B.2 (Star discrepancy bound)**.** *For the Kronecker sequence one has*

$$E_N = N D_N^* = \sup_{0 \leq a \leq 1} |S_N([0, a)) - Na| \leq 2 \sum_{k=0}^{m} a_{k+1}, \tag{59}$$

where m is the index such that $q_m \leq N < q_{m+1}$.

Proof. Apply Theorem B.1 to $W = [0, a)$ and take the supremum over $a \in [0, 1]$. □

Bounded type implies logarithmic mismatch. If α is badly approximable, the partial quotients are uniformly bounded, $a_i \leq A$. Then Corollary B.2 gives $E_N \leq 2A(m+1)$. Since $a_{k+1} \geq 1$ implies $q_{k+1} \geq q_k + q_{k-1}$, one has $q_k \geq F_k$ (Fibonacci), hence $m = O(\log N)$ and therefore

$$D_N^* = O\left(\frac{\log N}{N}\right), \qquad E_N = O(\log N). \tag{60}$$

Golden branch constants. For $\alpha = \varphi^{-1}$, all partial quotients satisfy $a_i = 1$ and $q_m = F_{m+1}$ (Fibonacci numbers). Then Eq. (55) becomes

$$D_N^* \;\leq\; \frac{m+1}{N} \qquad \text{whenever } F_{m+1} \leq N < F_{m+2}, \tag{61}$$

which makes the logarithmic growth of E_N explicit.

B.3 Rational slopes and linear mismatch

If $\alpha = p/q$ with $\gcd(p, q) = 1$, then $\{x_n\}$ takes values on a q-point lattice and is periodic with period q. For N that is a multiple of q, the multiset $\{x_1, \ldots, x_N\}$ consists of the same q points repeated equally often, hence its star discrepancy equals the star discrepancy of the underlying q-point set.

Lemma B.3 (Universal lower bound for 1D star discrepancy)**.** *Let $0 \leq y_1 \leq \cdots \leq y_q < 1$. Then the 1D star discrepancy of the q-point set satisfies*

$$D_q^* \geq \frac{1}{2q}. \tag{62}$$

Proof. For a sorted set, the star discrepancy admits the standard 1D formula

$$D_q^* = \max\left\{ \max_{1 \leq i \leq q} \left(\frac{i}{q} - y_i \right), \ \max_{1 \leq i \leq q} \left(y_i - \frac{i-1}{q} \right) \right\}.$$

If $D_q^* < 1/(2q)$, then for every i we would have

$$y_i > \frac{i}{q} - \frac{1}{2q} = \frac{2i-1}{2q} \quad \text{and} \quad y_i < \frac{i-1}{q} + \frac{1}{2q} = \frac{2i-1}{2q},$$

a contradiction. □

N	$E_N(\alpha = \varphi^{-1})$	$E_N(\alpha = \sqrt{2} - 1)$	$E_N(\alpha = 1/2)$
100	1.763207	1.374521	37.654321
300	1.821446	2.092946	112.962963
1000	1.793023	1.927121	376.543211
3000	2.345318	2.934988	1129.629633
10000	3.350444	3.706156	3765.432110
30000	2.823758	3.305305	11296.296330
100000	2.691072	2.855873	37654.321100

Table 1: **Representative accumulated mismatch.** The rational slope exhibits linear growth, while the irrational slopes remain at $O(\log N)$ scale (consistent with Corollary B.2).

Consequently, for $\alpha = p/q$ and every N divisible by q,

$$D_N^* \geq \frac{1}{2q}, \qquad E_N = N D_N^* \geq \frac{N}{2q}. \tag{63}$$

This is the arithmetic mechanism behind periodic phase locking and the collapse of accessible macrostates under fixed finite resolution.

Exact period-2 case $\alpha = 1/2$. Let $\alpha = 1/2$ and write $a := x_0 - \lfloor x_0 \rfloor \in [0, 1)$. Then the orbit visits the two points $\{a,\ a + 1/2 \bmod 1\}$, which can be written as $\{u, u + 1/2\}$ for a unique $u \in [0, 1/2)$. For even N, the empirical distribution assigns weight $1/2$ to each point, and a direct evaluation of the empirical CDF shows

$$D_N^* = \max\left(u, \frac{1}{2} - u\right), \qquad E_N = N \max\left(u, \frac{1}{2} - u\right). \tag{64}$$

This matches the exact linear fit observed in the toy experiment for $\alpha = 1/2$.

B.4 Entropy production proxies

In ASM the phase-friction entropy for a length-N window is

$$S_{\mathrm{pf}}(N) = k_B E_N. \tag{65}$$

If one interprets D_N^* as a per-step mismatch intensity, a natural scan-time proxy for the entropy production rate is

$$\frac{\mathrm{d}S_{\mathrm{pf}}}{\mathrm{d}\tau} \sim k_B\, D_N^*, \tag{66}$$

with the understanding that the right-hand side depends on the scale N at which coarse-graining is performed. The lapse rescaling $\mathrm{d}S/\mathrm{d}t = (\mathrm{d}S/\mathrm{d}\tau)(\mathrm{d}\tau/\mathrm{d}t)$ then yields Eq. (31) in the main text.

B.5 Numerical sanity checks (toy)

To make the arithmetic bounds tangible, Table 1 reports representative values of E_N at the phase offset $x_0 = 0.123456789$ for two irrational slopes and one rational slope. Empirical values fluctuate with N and with x_0 (because star discrepancy is anchored at 0), but the qualitative separation between logarithmic and linear regimes is robust.

Log-fit diagnostic (finite range). Fitting $E_N \approx A \log N + B$ over the N values in Table 1 yields

$$A_{\varphi^{-1}} \approx 0.196992, \quad B_{\varphi^{-1}} \approx 0.786635, \quad R^2 \approx 0.6337, \tag{67}$$

and

$$A_{\sqrt{2}-1} \approx 0.267471, \quad B_{\sqrt{2}-1} \approx 0.450037, \quad R^2 \approx 0.6460. \tag{68}$$

This fit is not a theorem; it is an operational diagnostic consistent with the $O(\log N)$ bound.

x_0-sensitivity (toy). At $N = 30000$, a uniform grid $x_0 \in \{0, \frac{1}{16}, \ldots, \frac{15}{16}\}$ gives:

slope	$\min E_N$	$\max E_N$	mean	std
φ^{-1}	2.421996	3.421996	2.629265	0.382915
$\sqrt{2}-1$	2.551632	3.551632	2.848136	0.339885
1/2	7500.000000	15000.000000	11250.000000	2296.396634

C Phase potential and the Newtonian limit

This appendix records the minimal variational closure used for the phase potential Φ and its weak-field interpretation.

C.1 Variational closure and Poisson equation

Let $\sigma(\mathbf{x})$ denote a coarse-grained mismatch density in three spatial dimensions, and define an effective source density $\rho_\Phi = \kappa_\Phi \sigma$ with coupling constant κ_Φ. We define the phase potential Φ as the stationary point of the quadratic functional

$$S_\Phi[\Phi; \sigma] := \int_{\mathbb{R}^3} \mathrm{d}^3\mathbf{x} \left(\frac{1}{8\pi} |\nabla \Phi|^2 + \kappa_\Phi \, \sigma \, \Phi \right), \tag{69}$$

with appropriate boundary conditions (e.g. $\Phi \to 0$ at spatial infinity). The Euler–Lagrange equation is the Poisson equation

$$\Delta \Phi(\mathbf{x}) = 4\pi \, \rho_\Phi(\mathbf{x}) = 4\pi \kappa_\Phi \, \sigma(\mathbf{x}). \tag{70}$$

The associated phase-pressure field is

$$\mathbf{P}_\Phi(\mathbf{x}) = -\nabla \Phi(\mathbf{x}). \tag{71}$$

C.2 Point-source solution

On $\mathbb{R}^3$ with decay boundary condition at infinity, the Green's function for Δ gives

$$\Phi(\mathbf{x}) = -\int_{\mathbb{R}^3} \frac{\rho_\Phi(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} \, \mathrm{d}^3\mathbf{y}. \tag{72}$$

Equivalently, $\Delta(1/|\mathbf{x}|) = -4\pi\delta(\mathbf{x})$ [51].

Mismatch charge and exterior monopole. Define the total mismatch charge

$$Q_\sigma := \int_{\mathbb{R}^3} \sigma(\mathbf{y}) \, \mathrm{d}^3\mathbf{y}. \tag{73}$$

For compactly supported σ, the far-field expansion is

$$\Phi(r) = -\frac{M_\Phi}{r} + O(r^{-2}), \qquad M_\Phi := \kappa_\Phi \, Q_\sigma. \tag{74}$$

Thus the monopole coefficient is fixed by the total mismatch charge Q_σ and the calibration constant κ_Φ. For a localized isolated source $\sigma(\mathbf{x}) = Q\,\delta^{(3)}(\mathbf{x})$ one obtains

$$\Phi(r) = -\kappa_\Phi \frac{Q}{r}, \qquad r = |\mathbf{x}|. \tag{75}$$

Then

$$\mathbf{P}_\Phi(r) = -\nabla\Phi(r) = -\kappa_\Phi \frac{Q}{r^2}\,\hat{\mathbf{r}}, \tag{76}$$

and $M_\Phi = \kappa_\Phi Q$ as expected. In the Newtonian limit, $\Phi_N(r) = -GM_g/r$ for gravitational mass M_g, so matching the monopole term yields the calibration condition

$$\kappa_\Phi\, Q_\sigma = G\, M_g. \tag{77}$$

Equivalently, one may define an effective mass density $\rho_\sigma := (\kappa_\Phi/G)\,\sigma$ so that $\Delta\Phi = 4\pi G\rho_\sigma$ and $M_g = \int \rho_\sigma$. In this effective template, gravity is interpreted as a deterministic response to readout mismatch density rather than a fundamental interaction.

C.3 Remarks

The Poisson closure is a macroscopic effective choice consistent with locality and the existence of a scalar potential sourced by a coarse-grained density. In the full HPA–Ω program, σ is expected to be computable from explicit microscopic readout protocols and local implementation constraints; the present paper uses it as the minimal interface object enabling thermodynamic and phenomenological predictions.

C.4 Rigidity of the quadratic closure

The functional $S_\Phi[\Phi;\sigma]$ in Eq. (69) can be viewed as a rigidity statement: under mild symmetry assumptions, the Poisson closure is the unique local quadratic response law for a shift-symmetric scalar potential.

Proposition C.1 (Uniqueness of the local quadratic action)**.** *Assume an effective macroscopic scalar potential Φ is defined only up to an additive constant (shift symmetry), and that its stationary response to a source density σ is governed by a local action functional that is (i) quadratic in Φ and its first derivatives, (ii) translation and rotation invariant in $\mathbb{R}^3$, and (iii) involves no higher than first derivatives of Φ in the bulk. Then, up to an overall scale and boundary terms, the unique bulk quadratic form is $\int |\nabla\Phi|^2$, and the unique linear coupling to σ is $\int \sigma\,\Phi$. Consequently the Euler–Lagrange equation is Poisson:*

$$\Delta\Phi = 4\pi\kappa_\Phi\,\sigma, \tag{78}$$

with κ_Φ set by calibration.

Proof. By shift symmetry, no bulk term proportional to Φ^2 is allowed, and Φ can enter only through derivatives or linearly through the source coupling. The most general translation- and rotation-invariant quadratic bulk form built from first derivatives is

$$\int_{\mathbb{R}^3} a\,\partial_i\Phi\,\partial_i\Phi\,\mathrm{d}^3x$$

for some constant $a > 0$ (Einstein summation). Cross-terms differ only by boundary contributions after integration by parts. The only local linear source coupling consistent with symmetries is $\int b\,\sigma\,\Phi\,\mathrm{d}^3x$. Varying yields $-2a\,\Delta\Phi + b\,\sigma = 0$, which is equivalent to Poisson after rescaling and fixing the normalization to the conventional 4π factor [33]. □

Signed discrepancy vs. mismatch charge. The signed equidistribution defect $\mu_N - \mu$ has zero total charge and therefore cannot generate a monopole $1/r$ term. The σ used in this paper is not a signed measure; it is a nonnegative coarse-grained *cost density* (phase-friction production), which can carry nonzero total charge in isolated defect sectors.

Flux characterization. By the divergence theorem and $\Delta\Phi = 4\pi\kappa_\Phi\sigma$,

$$\int_{S_R} \mathbf{P}_\Phi \cdot \mathrm{d}\mathbf{S} = -\int_{B_R} \Delta\Phi \, \mathrm{d}^3\mathbf{x} = -4\pi\kappa_\Phi \int_{B_R} \sigma \, \mathrm{d}^3\mathbf{x}, \tag{79}$$

so in the limit $R \to \infty$ one has

$$M_\Phi = \kappa_\Phi Q_\sigma = -\frac{1}{4\pi} \lim_{R\to\infty} \int_{S_R} \mathbf{P}_\Phi \cdot \mathrm{d}\mathbf{S}. \tag{80}$$

D Geometric Landauer principle and impedance

This appendix records a minimal formulation of the geometric Landauer bound used in the main text.

D.1 Log-cost additivity

In the scan–projection semantics, operational "success" is naturally multiplicative: if two independent constraints have weights $w_1, w_2 \in (0, 1]$, then their joint weight is $w = w_1 w_2$. Defining a log-cost

$$C := -\log w, \tag{81}$$

one obtains additivity

$$C = C_1 + C_2, \qquad C_i := -\log w_i. \tag{82}$$

This is the structural reason entropies appear as logarithms in ASM.

D.2 Landauer bound and geometric overhead

Landauer's principle bounds the minimal work required to erase one bit at temperature T by [25, 26]

$$W_{\text{erase}} \geq k_B T \ln 2. \tag{83}$$

In HPA–Ω, the relevant operational temperature is the computational temperature T_c, and erasure must be implemented on a constrained local network with routing overhead κ (or equivalently lapse $\mathcal{N} = \kappa_0/\kappa$). This motivates a refined bound

$$W_{\text{erase}} \geq k_B T_c \ln 2 + Z_{\text{geom}}. \tag{84}$$

D.3 A convenient impedance parameterization

The term Z_{geom} captures extra work required by geometric constraints such as routing, locality, and impedance. A convenient choice consistent with log-cost additivity and with monotonic increase of cost under slowdown is forced (up to a single dimensionless constant) by a simple functional equation.

Proposition D.1 (Log-overhead form)**.** *Let $s := \kappa/\kappa_0 \geq 1$ denote the dimensionless routing-overhead factor and assume that the geometric impedance term can be written as*

$$Z_{\text{geom}}(s) = k_B T_c \, F(s), \tag{85}$$

where $F : [1, \infty) \to \mathbb{R}$ satisfies the normalization $F(1) = 0$ and the log-additivity condition

$$F(s_1 s_2) = F(s_1) + F(s_2) \qquad (s_1, s_2 \geq 1), \tag{86}$$

expressing the multiplicative composition of serial overhead factors. If F is measurable (or continuous) on $[1, \infty)$, then there exists a constant $\zeta_{\rm geom} \geq 0$ such that

$$Z_{\rm geom}(s) = \zeta_{\rm geom}\, k_B T_c \log s \;=\; \zeta_{\rm geom}\, k_B T_c \log\!\left(\frac{\kappa}{\kappa_0}\right) \;=\; \zeta_{\rm geom}\, k_B T_c \log\!\left(\frac{1}{\mathcal{N}}\right). \tag{87}$$

Proof. Define $g : \mathbb{R}_{\geq 0} \to \mathbb{R}$ by $g(x) := F(\mathrm{e}^x)$. Then $g(x + y) = g(x) + g(y)$ for all $x, y \geq 0$. Measurability (or continuity) implies the standard Cauchy conclusion $g(x) = \zeta_{\rm geom} x$ for some constant $\zeta_{\rm geom}$, hence $F(s) = \zeta_{\rm geom} \log s$. □

This form makes the semantics transparent:

- larger routing overhead κ (smaller $\mathcal{N}$) increases the minimal work cost,
- serial routing segments contribute additively in log-overhead.

The paper uses $Z_{\rm geom}$ as an interface term: the dimensionless constant $\zeta_{\rm geom}$ can be fixed once an explicit compilation-depth model or a benchmark calibration protocol is chosen.

E Reproducible toy experiments (Python)

This appendix contains reference implementations for Section 9. They require only Python 3 (no third-party dependencies). A minimal requirement file is provided in `requirements.txt`.

E.1 Experiment A: star discrepancy and accumulated mismatch E_N

```python
"""
Experiment A: star discrepancy and accumulated mismatch for rotation sequences.

Pure-Python (no third-party dependencies) reference implementation.

We compare accumulated mismatch E_N = N * D_N^* for:
- an irrational slope (golden branch),
- another irrational slope (sqrt(2) - 1),
- a rational slope (1/2) as a simple phase-locking / periodic case.

The output illustrates compatibility with O(log N) for badly approximable
irrationals, and linear growth for rational slopes.
"""

from __future__ import annotations

import math


def rotation_points(alpha: float, N: int, x0: float = 0.0) -> list[float]:
    pts: list[float] = []
    x = float(x0)
    a = float(alpha)
    for n in range(1, N + 1):
        x = x0 + n * a
        pts.append(x - math.floor(x))
    return pts
```

```python
def star_discrepancy(points: list[float]) -> float:
    """1D star discrepancy for points in [0, 1)."""
    x = sorted(points)
    N = len(x)
    invN = 1.0 / float(N)

    d1 = 0.0
    d2 = 0.0
    for idx, xi in enumerate(x, start=1):
        i_over_N = idx * invN
        im1_over_N = (idx - 1) * invN
        d1 = max(d1, abs(i_over_N - xi))
        d2 = max(d2, abs(xi - im1_over_N))
    return max(d1, d2)


def accumulated_mismatch(alpha: float, N: int, x0: float = 0.0) -> float:
    pts = rotation_points(alpha, N, x0=x0)
    D = star_discrepancy(pts)
    return float(N) * D


def unique_points_count(alpha: float, N: int, x0: float = 0.0, tol: float = 1e-12) ->
↪   int:
    pts = rotation_points(alpha, N, x0=x0)
    quant = set()
    inv_tol = 1.0 / tol
    for p in pts:
        q = int(round(p * inv_tol))
        quant.add(q)
    return len(quant)

def linfit(xs: list[float], ys: list[float]):
    """Least squares fit y = a x + b and R^2."""
    n = len(xs)
    mx = sum(xs) / n
    my = sum(ys) / n
    sxx = sum((x - mx) ** 2 for x in xs)
    sxy = sum((x - mx) * (y - my) for x, y in zip(xs, ys))
    a = sxy / sxx if sxx != 0 else float("nan")
    b = my - a * mx
    sst = sum((y - my) ** 2 for y in ys)
    sse = sum((y - (a * x + b)) ** 2 for x, y in zip(xs, ys))
    r2 = 1.0 - sse / sst if sst != 0 else float("nan")
    return a, b, r2


def stats(vals: list[float]):
    n = len(vals)
    mean = sum(vals) / n
    var = sum((v - mean) ** 2 for v in vals) / n
    return min(vals), max(vals), mean, math.sqrt(var)


def main() -> None:
    alpha_golden = (math.sqrt(5.0) - 1.0) / 2.0
    alpha_sqrt2 = math.sqrt(2.0) - 1.0
```

```
alpha_rational = 1.0 / 2.0

Ns = [100, 300, 1_000, 3_000, 10_000, 30_000, 100_000]
x0 = 0.123456789

print("N, E_N(golden), E_N(sqrt2-1), E_N(rational=1/2)")
Eg_list: list[float] = []
Es_list: list[float] = []
for N in Ns:
    Eg = accumulated_mismatch(alpha_golden, N, x0=x0)
    Es = accumulated_mismatch(alpha_sqrt2, N, x0=x0)
    Er = accumulated_mismatch(alpha_rational, N, x0=x0)
    Eg_list.append(Eg)
    Es_list.append(Es)
    print(f"{N:>8d}  {Eg:>12.6f}  {Es:>12.6f}  {Er:>14.6f}")

print("\nCompatibility check: E_N/log N (smaller and slowly varying suggests O(log
↪  N))")
print("N, Eg/logN, Es/logN, Er/logN")
for N in Ns:
    logN = math.log(float(N))
    Eg = accumulated_mismatch(alpha_golden, N, x0=x0) / logN
    Es = accumulated_mismatch(alpha_sqrt2, N, x0=x0) / logN
    Er = accumulated_mismatch(alpha_rational, N, x0=x0) / logN
    print(f"{N:>8d}  {Eg:>12.6f}  {Es:>12.6f}  {Er:>14.6f}")

print("\nPeriodicity sanity check (unique points count for N=2000):")
Np = 2000
ug = unique_points_count(alpha_golden, Np, x0=x0)
us = unique_points_count(alpha_sqrt2, Np, x0=x0)
ur = unique_points_count(alpha_rational, Np, x0=x0)
print(f"golden:  {ug} unique points (should be large)")
print(f"sqrt2-1: {us} unique points (should be large)")
print(f"1/2:     {ur} unique points (should be 2)")

# Explicit continued-fraction envelope bounds for two standard badly-approximable
↪  slopes.
# Using the bound D_N^* <= (1 + sum_{i=1}^m a_i)/N for q_m <= N < q_{m+1},
# where a_i are continued-fraction partial quotients and q_m are denominators.
#
# For golden: a_i=1, q_m=F_{m+1}, so E_N <= 1+m when F_{m+1}<=N<F_{m+2}.
# For sqrt(2)-1: a_i=2, q_m follow Pell-type recurrence q_{m+1}=2q_m+q_{m-1}.
def fib_index_for_N(N: int) -> int:
    # Return m such that F_{m+1} <= N < F_{m+2}, with F_1=1,F_2=1.
    f_prev, f = 1, 1  # F_1, F_2
    m = 1  # corresponds to F_{m+1}=F_2 initially
    while True:
        f_next = f_prev + f  # next Fibonacci
        if f_next > N:
            return m
        f_prev, f = f, f_next
        m += 1

def pell_index_for_N(N: int) -> int:
    # Denominators for [0;2,2,2,...] satisfy q_0=1,q_1=2,q_{m+1}=2q_m+q_{m-1}.
    q_prev, q = 1, 2
    m = 1
    while True:
```

```
            q_next = 2 * q + q_prev
            if q_next > N:
                return m
            q_prev, q = q, q_next
            m += 1

    print("\nExplicit envelope upper bounds from continued fractions (via
    ↪  Kuipers--Niederreiter):")
    print("N, E_N(golden), bound_golden, E_N(sqrt2-1), bound_sqrt2-1")
    for N in Ns:
        Eg = accumulated_mismatch(alpha_golden, N, x0=x0)
        Es = accumulated_mismatch(alpha_sqrt2, N, x0=x0)

        m_g = fib_index_for_N(N)
        bound_g = 1.0 + float(m_g)  # 1 + sum a_i = 1 + m (all a_i=1)

        m_s = pell_index_for_N(N)
        bound_s = 1.0 + 2.0 * float(m_s)  # 1 + sum a_i = 1 + 2m (all a_i=2)

        print(f"{N:>8d}  {Eg:>12.6f}  {bound_g:>12.6f}  {Es:>12.6f}
        ↪  {bound_s:>12.6f}")

    print("\nFit diagnostic for irrationals: E_N ~ A log N + B (finite range)")
    xs = [math.log(float(N)) for N in Ns]
    A_g, B_g, r2_g = linfit(xs, Eg_list)
    A_s, B_s, r2_s = linfit(xs, Es_list)
    print(f"golden:   A={A_g:.6f}, B={B_g:.6f}, R2={r2_g:.6f}")
    print(f"sqrt2-1:  A={A_s:.6f}, B={B_s:.6f}, R2={r2_s:.6f}")

    print("\nPhase-offset sensitivity at N=30000 over x0 in {0,1/16,...,15/16}:")
    N0 = 30000
    x0s = [i / 16.0 for i in range(16)]
    for name, alpha in [("golden", alpha_golden), ("sqrt2-1", alpha_sqrt2), ("1/2",
    ↪  alpha_rational)]:
        vals = [accumulated_mismatch(alpha, N0, x0=float(xx)) for xx in x0s]
        mn, mx, mean, sd = stats(vals)
        print(f"{name:>6s}: min={mn:.6f} max={mx:.6f} mean={mean:.6f} std={sd:.6f}")


if __name__ == "__main__":
    main()
```

E.2 Experiment B: computational lapse rescaling of entropy flow

```
"""
Experiment B: computational lapse rescaling of entropy flow.

Pure-Python (no third-party dependencies) reference implementation.

We treat the per-scan-step mismatch density sigma as a toy proxy given by
star discrepancy D_N^* for a large window N. The externally observed entropy
production rate is then:

  dS/dt = k_B * sigma * lapse.

This script demonstrates linear scaling in the lapse factor.
```

```python
"""

from __future__ import annotations

import math


def rotation_points(alpha: float, N: int, x0: float = 0.0) -> list[float]:
    pts: list[float] = []
    a = float(alpha)
    for n in range(1, N + 1):
        x = x0 + n * a
        pts.append(x - math.floor(x))
    return pts


def star_discrepancy(points: list[float]) -> float:
    x = sorted(points)
    N = len(x)
    invN = 1.0 / float(N)

    d1 = 0.0
    d2 = 0.0
    for idx, xi in enumerate(x, start=1):
        i_over_N = idx * invN
        im1_over_N = (idx - 1) * invN
        d1 = max(d1, abs(i_over_N - xi))
        d2 = max(d2, abs(xi - im1_over_N))
    return max(d1, d2)


def main() -> None:
    kB = 1.0
    alpha_golden = (math.sqrt(5.0) - 1.0) / 2.0
    N = 100_000
    x0 = 0.123456789

    D = star_discrepancy(rotation_points(alpha_golden, N, x0=x0))
    sigma = D  # per-scan-step mismatch proxy

    print("Using sigma := D_N^* for the golden branch as a toy proxy.")
    print(f"N={N}, D_N^*={D:.8e}")

    lapses = [1.0, 0.5, 0.2, 0.1, 0.02]
    print("\nToy rescaling: dS/dt = k_B * sigma * lapse")
    baseline = kB * sigma * lapses[0]
    for L in lapses:
        rate = kB * sigma * L
        ratio = rate / baseline if baseline != 0.0 else float("nan")
        print(f"lapse={L:>6.2f}  dS/dt={rate:.8e}  ratio={ratio:.4f}")


if __name__ == "__main__":
    main()
```

E.3 Experiment C: least-squares fits for mismatch templates

```
"""
Experiment C: least-squares fits for mismatch-growth templates.

Pure-Python (no third-party dependencies) reference implementation.

We fit:
  (i) irrationals:  E_N = a * log N + b
 (ii) rationals:   E_N = c * N + d

The script prints fitted coefficients and R^2, and can emit a LaTeX table.

Usage:
  python3 scripts/experiment_c_fit_tables.py
  python3 scripts/experiment_c_fit_tables.py --latex
"""

from __future__ import annotations

import math
import sys
from dataclasses import dataclass


def rotation_points(alpha: float, N: int, x0: float = 0.0) -> list[float]:
    pts: list[float] = []
    a = float(alpha)
    for n in range(1, N + 1):
        x = x0 + n * a
        pts.append(x - math.floor(x))
    return pts


def star_discrepancy(points: list[float]) -> float:
    x = sorted(points)
    N = len(x)
    invN = 1.0 / float(N)

    d1 = 0.0
    d2 = 0.0
    for idx, xi in enumerate(x, start=1):
        i_over_N = idx * invN
        im1_over_N = (idx - 1) * invN
        d1 = max(d1, abs(i_over_N - xi))
        d2 = max(d2, abs(xi - im1_over_N))
    return max(d1, d2)


def accumulated_mismatch(alpha: float, N: int, x0: float = 0.0) -> float:
    pts = rotation_points(alpha, N, x0=x0)
    D = star_discrepancy(pts)
    return float(N) * D


@dataclass(frozen=True)
class Fit:
    a: float
    b: float
```

```
    r2: float


def linear_regression(x: list[float], y: list[float]) -> Fit:
    n = len(x)
    if n != len(y) or n < 2:
        raise ValueError("Need at least two data points with matched lengths.")

    sx = sum(x)
    sy = sum(y)
    sxx = sum(v * v for v in x)
    sxy = sum(xi * yi for xi, yi in zip(x, y))

    denom = n * sxx - sx * sx
    if denom == 0.0:
        raise ValueError("Degenerate x values (cannot fit).")

    a = (n * sxy - sx * sy) / denom
    b = (sy - a * sx) / n

    y_mean = sy / n
    ss_tot = sum((yi - y_mean) ** 2 for yi in y)
    ss_res = sum((yi - (a * xi + b)) ** 2 for xi, yi in zip(x, y))
    r2 = 1.0 - (ss_res / ss_tot if ss_tot != 0.0 else 0.0)

    return Fit(a=a, b=b, r2=r2)


def as_latex_row(label: str, model: str, fit: Fit) -> str:
    return (
        f"{label} & {model} & {fit.a:.6f} & {fit.b:.6f} & {fit.r2:.6f} \\\\"
    )


def running_max(values: list[float]) -> list[float]:
    out: list[float] = []
    m = -float("inf")
    for v in values:
        if v > m:
            m = v
        out.append(m)
    return out


def main() -> None:
    alpha_golden = (math.sqrt(5.0) - 1.0) / 2.0
    alpha_sqrt2 = math.sqrt(2.0) - 1.0
    alpha_rational = 1.0 / 2.0

    Ns = [100, 300, 1_000, 3_000, 10_000, 30_000, 100_000]
    x0 = 0.123456789

    def series(alpha: float) -> list[float]:
        return [accumulated_mismatch(alpha, N, x0=x0) for N in Ns]

    Eg = series(alpha_golden)
    Es = series(alpha_sqrt2)
    Er = series(alpha_rational)
```

```
    x_log = [math.log(float(N)) for N in Ns]
    x_lin = [float(N) for N in Ns]

    Eg_env = running_max(Eg)
    Es_env = running_max(Es)

    fit_g = linear_regression(x_log, Eg_env)
    fit_s = linear_regression(x_log, Es_env)
    fit_r = linear_regression(x_lin, Er)

    print("Fit templates:")
    print("  irrationals: E_N = a * log N + b")
    print("  rationals:   E_N = c * N + d\n")

    print(f"Initial phase x0 = {x0}")
    print(f"N samples = {Ns}\n")

    print("Golden branch (envelope fit):")
    print(f"  a={fit_g.a:.6f}, b={fit_g.b:.6f}, R^2={fit_g.r2:.6f}")
    print("sqrt(2)-1 (envelope fit):")
    print(f"  a={fit_s.a:.6f}, b={fit_s.b:.6f}, R^2={fit_s.r2:.6f}")
    print("Rational 1/2:")
    print(f"  c={fit_r.a:.6f}, d={fit_r.b:.6f}, R^2={fit_r.r2:.6f}\n")

    want_latex = "--latex" in sys.argv[1:]
    if want_latex:
        print("% LaTeX table rows (a,b,R^2):")
        print(as_latex_row(r"$\\alpha=\\varphi^{-1}$", r"$E_N^{\\uparrow}=a\\log
        ↪  N+b$", fit_g))
        print(as_latex_row(r"$\\alpha=\\sqrt{2}-1$", r"$E_N^{\\uparrow}=a\\log N+b$",
        ↪  fit_s))
        print(as_latex_row(r"$\\alpha=1/2$", r"$E_N=cN+d$", fit_r))


if __name__ == "__main__":
    main()
```

E.4 Experiment D: $1/f$ spectrum from Fibonacci/geometric ladders

```
"""
Experiment D: 1/f spectrum from Fibonacci/geometric relaxation ladders.

Pure-Python (no third-party dependencies) reference implementation.

We construct a ladder spectrum
  S(f) = sum_k w * tau_k / (1 + (2*pi*f*tau_k)^2)
with:
  (A) geometric times tau_k = tau0 * r^k
  (B) Fibonacci times  tau_k = tau0 * F_{k+1}

and fit log S vs log f over a mid-band to estimate the slope and R^2.
"""

from __future__ import annotations
```

```python
import math
from dataclasses import dataclass
import sys


@dataclass(frozen=True)
class Fit:
    slope: float
    intercept: float
    r2: float


def linfit(xs: list[float], ys: list[float]) -> Fit:
    n = len(xs)
    mx = sum(xs) / n
    my = sum(ys) / n
    sxx = sum((x - mx) ** 2 for x in xs)
    sxy = sum((x - mx) * (y - my) for x, y in zip(xs, ys))
    slope = sxy / sxx if sxx != 0.0 else float("nan")
    intercept = my - slope * mx
    sst = sum((y - my) ** 2 for y in ys)
    sse = sum((y - (slope * x + intercept)) ** 2 for x, y in zip(xs, ys))
    r2 = 1.0 - sse / sst if sst != 0.0 else float("nan")
    return Fit(slope=slope, intercept=intercept, r2=r2)


def fib_times(K: int, tau0: float = 1.0) -> list[float]:
    # Use F_{k+1} with F_0=0,F_1=1.
    f0, f1 = 0, 1
    out: list[float] = []
    for _ in range(K + 1):
        f0, f1 = f1, f0 + f1  # now f0 = F_{k+1}
        out.append(tau0 * float(f0))
    return out


def geometric_times(K: int, r: float, tau0: float = 1.0) -> list[float]:
    return [tau0 * (r ** k) for k in range(K + 1)]


def ladder_spectrum(freqs: list[float], taus: list[float], w: float = 1.0) ->
↪   list[float]:
    out: list[float] = []
    for f in freqs:
        omega = 2.0 * math.pi * f
        s = 0.0
        for tau in taus:
            x = omega * tau
            s += w * tau / (1.0 + x * x)
        out.append(s)
    return out


def logspace(f_min: float, f_max: float, n: int) -> list[float]:
    if f_min <= 0.0 or f_max <= 0.0 or f_max <= f_min:
        raise ValueError("Need 0 < f_min < f_max.")
    a = math.log10(f_min)
    b = math.log10(f_max)
```

```
    step = (b - a) / (n - 1)
    return [10.0 ** (a + i * step) for i in range(n)]


def fit_band(freqs: list[float], S: list[float], f_lo: float, f_hi: float) -> Fit:
    xs: list[float] = []
    ys: list[float] = []
    for f, s in zip(freqs, S):
        if f_lo <= f <= f_hi and s > 0.0:
            xs.append(math.log(f))
            ys.append(math.log(s))
    if len(xs) < 5:
        raise ValueError("Not enough points in fit band.")
    return linfit(xs, ys)


def main() -> None:
    phi = (1.0 + math.sqrt(5.0)) / 2.0
    tau0 = 1.0
    K = 24  # gives tau_max ~ phi^K or F_{K+1} ~ phi^{K+1}/sqrt(5)
    w = 1.0

    taus_geo = geometric_times(K, r=phi, tau0=tau0)
    taus_fib = fib_times(K, tau0=tau0)

    f_min = 1e-4
    f_max = 1e2
    freqs = logspace(f_min, f_max, 500)

    S_geo = ladder_spectrum(freqs, taus_geo, w=w)
    S_fib = ladder_spectrum(freqs, taus_fib, w=w)

    # Choose a mid-band that satisfies the asymptotic conditions in Proposition
    ↪  app:fibonacci_1f.
    # We enforce omega*tau_min << 1 and omega*tau_max >> 1 with conservative margins.
    tau_min_geo, tau_max_geo = min(taus_geo), max(taus_geo)
    tau_min_fib, tau_max_fib = min(taus_fib), max(taus_fib)
    tau_min = min(tau_min_geo, tau_min_fib)
    tau_max = max(tau_max_geo, tau_max_fib)

    f_lo = 10.0 / (2.0 * math.pi * tau_max)   # omega*tau_max >= 10
    f_hi = 0.1 / (2.0 * math.pi * tau_min)    # omega*tau_min <= 0.1
    fit_geo = fit_band(freqs, S_geo, f_lo=f_lo, f_hi=f_hi)
    fit_fib = fit_band(freqs, S_fib, f_lo=f_lo, f_hi=f_hi)

    print("1/f ladder spectrum fit (log-log): log S = slope * log f + intercept")
    print(f"Fit band: f in [{f_lo}, {f_hi}]")
    print(f"Geometric r=phi: slope={fit_geo.slope:.4f}, R^2={fit_geo.r2:.6f}")
    print(f"Fibonacci:       slope={fit_fib.slope:.4f}, R^2={fit_fib.r2:.6f}")

    # Theoretical slope is -1, and the prefactor for the continuous-log approximation
    ↪  is ~ w/(4 ln r).
    pref = w / (4.0 * math.log(phi))
    print(f"Theory (continuous log-uniform, r=phi): slope=-1, prefactor~={pref:.6f}
    ↪  (units of S*f)")

    if "--latex" in sys.argv[1:]:
        print("% LaTeX table rows: model & K & fit-band & slope & R^2")
```

```
        band = f"[{f_lo:.3e},{f_hi:.3e}]"
        print(f"Geometric ($r=\\varphi$) & {K} & {band} & {fit_geo.slope:.6f} &
        ↪  {fit_geo.r2:.6f} \\\\")
        print(f"Fibonacci ($\\tau_k\\propto F_k$) & {K} & {band} & {fit_fib.slope:.6f}
        ↪  & {fit_fib.r2:.6f} \\\\")


if __name__ == "__main__":
    main()
```

F Quantitative fits for mismatch templates

This appendix reports least-squares fits for the toy mismatch-growth templates discussed in Section 9. A practical subtlety is that E_N for Kronecker sequences exhibits number-theoretic oscillations and is not monotone in N. Since the theoretical statements are envelope bounds (e.g. $E_N = O(\log N)$ for badly approximable slopes), we fit an empirical upper envelope

$$E_N^{\uparrow} := \max_{M \le N} E_M, \tag{88}$$

approximated on a discrete sample of N values by the running maximum.

Dataset. We use initial phase $x_0 = 0.123456789$ and sample sizes

$$N \in \{100, 300, 10^3, 3 \times 10^3, 10^4, 3 \times 10^4, 10^5\}.$$

The fitted values below are produced by the pure-Python script `scripts/experiment_c_fit_tables.py` (Appendix E).

Slope	Model	a	b	R^2
$\alpha = \varphi^{-1}$	$E_N^{\uparrow} = a \log N + b$	0.290129	0.211636	0.857333
$\alpha = \sqrt{2} - 1$	$E_N^{\uparrow} = a \log N + b$	0.366607	-0.144242	0.904830
$\alpha = 1/2$	$E_N = cN + d$	0.376543	0.000000	1.000000

Table 2: Toy-model fits for mismatch templates. For irrational slopes we fit the empirical envelope $E_N^{\uparrow}$ to the logarithmic template; for the rational slope the mismatch is exactly linear (Appendix B).

Provable envelope coefficients from continued fractions. Appendix B, Eq. (55) implies an explicit envelope bound of the form

$$E_N \le a_{\text{ub}}(\alpha) \log N + O(1)$$

for badly approximable slopes. For the golden branch $\alpha = \varphi^{-1}$ (all partial quotients $a_i = 1$ and $q_m = F_{m+1}$) one obtains

$$a_{\text{ub}}(\varphi^{-1}) = \frac{1}{\log \varphi} \approx 2.078087.$$

For $\alpha = \sqrt{2} - 1 = [0; 2, 2, 2, \dots]$ (all partial quotients $a_i = 2$ and q_m growing at rate $1 + \sqrt{2}$) one obtains

$$a_{\text{ub}}(\sqrt{2} - 1) = \frac{2}{\log(1 + \sqrt{2})} \approx 2.269185.$$

These are rigorous but not optimized; they provide a non-asymptotic calibration scale against which the fitted envelope coefficients a may be compared.

G Fibonacci hierarchy and $1/f$ spectra

This appendix closes the $1/f$ claim as a quantitative consequence of a Fibonacci/Zeckendorf scale hierarchy combined with a standard spectral superposition law.

G.1 Log-uniform scaling from Fibonacci times

Let $(F_k)_{k\geq 0}$ denote Fibonacci numbers with $F_0 = 0$, $F_1 = 1$, $F_{k+1} = F_k + F_{k-1}$. The closed form

$$F_k = \frac{\varphi^k - \hat{\varphi}^k}{\sqrt{5}}, \qquad \hat{\varphi} = -\varphi^{-1}, \tag{89}$$

implies the asymptotic ratio $F_{k+1}/F_k \to \varphi$ and therefore the log-spacing

$$\log F_{k+1} - \log F_k \to \log \varphi. \tag{90}$$

Hence Fibonacci times are asymptotically geometric: they form an approximately uniform lattice in $\log \tau$.

G.2 Discrete log-uniform mixtures yield $1/f$

Consider a family of relaxation-time modes indexed by k with time constants

$$\tau_k = \tau_0\, r^k, \qquad r > 1, \tag{91}$$

and mode weights $w_k \geq 0$. A standard Lorentzian single-mode spectrum takes the form

$$S_k(\omega) = w_k \frac{\tau_k}{1 + (\omega\tau_k)^2}, \tag{92}$$

which captures (up to normalization) the power spectrum of a wide class of exponentially relaxing fluctuations (e.g. random-telegraph or OU-type components) [40, 41].

Proposition G.1 (Geometric relaxation ladder implies $1/\omega$ plateau)**.** *Assume $w_k \equiv w$ (scale-equipartition across log-time). Define the ladder spectrum*

$$S_K(\omega) = \sum_{k=0}^{K} w \frac{\tau_0 r^k}{1 + (\omega\tau_0 r^k)^2}. \tag{93}$$

Then for intermediate frequencies satisfying

$$\omega\tau_0 \ll 1 \ll \omega\tau_0 r^K, \tag{94}$$

one has the asymptotic approximation

$$S_K(\omega) = \frac{w}{\omega \log r}\left(\frac{\pi}{2} + o(1)\right), \tag{95}$$

and therefore in frequency $f = \omega/(2\pi)$,

$$S_K(f) = \frac{w}{4f\, \log r}\left(1 + o(1)\right). \tag{96}$$

Proof. Approximate the sum by an integral over $\log \tau$. Since $\tau_k = \tau_0 r^k$, the spacing in $\log \tau$ is constant $\Delta = \log r$, so

$$S_K(\omega) \approx \frac{w}{\Delta}\int_{\tau_0}^{\tau_0 r^K} \frac{\tau}{1 + (\omega\tau)^2}\,\frac{\mathrm{d}\tau}{\tau} = \frac{w}{\omega\Delta}\left[\arctan(\omega\tau)\right]_{\tau_0}^{\tau_0 r^K}.$$

Under $\omega\tau_0 \ll 1 \ll \omega\tau_0 r^K$, the bracket tends to $\pi/2$, yielding the stated approximation. □

Corollary G.2 (Robustness to nonuniform weights)**.** *Assume the weights are bounded,* $w_- \le w_k \le w_+$ *for all* k*. Under the same intermediate-frequency condition* $\omega\tau_0 \ll 1 \ll \omega\tau_0 r^K$*, the ladder spectrum satisfies*

$$\frac{w_-}{4f\,\log r}\,(1+o(1)) \;\le\; S_K(f) \;\le\; \frac{w_+}{4f\,\log r}\,(1+o(1))\,, \tag{97}$$

and therefore exhibits a robust $1/f$ *mid-band. The weights affect only the prefactor, which is set by the effective log-scale average of* w_k *across the contributing ladder range.*

Proof. Each summand in S_K is nonnegative and linear in w_k, so termwise bounds give

$$\sum_{k=0}^{K} w_- \frac{\tau_0 r^k}{1+(\omega\tau_0 r^k)^2} \le S_K(\omega) \le \sum_{k=0}^{K} w_+ \frac{\tau_0 r^k}{1+(\omega\tau_0 r^k)^2}.$$

Applying Proposition G.1 to the constant-weight ladders yields the stated bounds after converting ω to $f = \omega/(2\pi)$. □

Fibonacci/Zeckendorf specialization. Taking $r = \varphi$ and $\tau_k \propto F_k$ yields the same intermediate scaling because $F_{k+1}/F_k \to \varphi$. Thus a multi-scale readout that aggregates approximately equal-weight relaxation contributions across Zeckendorf depth predicts a $1/f$ band with an explicit prefactor proportional to $(\log\varphi)^{-1}$.

G.3 Numerical verification (toy)

Appendix E includes a pure-Python script that constructs $S_K(f)$ for both geometric and Fibonacci ladders, performs a log–log linear fit over an automatically selected mid-band, and reports the fitted spectral slope and R^2. Representative fit parameters (fit band, slope, R^2) are reported in Appendix H.

H $1/f$ spectrum fits (toy)

This appendix reports the fitted spectral slopes for the ladder spectra constructed in Appendix G. The results are produced by `scripts/experiment_d_1f_spectrum.py -latex` (Appendix E), which selects a fit band satisfying the asymptotic conditions in Proposition G.1.

Model	K	Fit band $[f_{\min}, f_{\max}]$	slope	R^2
Geometric ($r = \varphi$)	24	$[1.535 \times 10^{-5}, 1.592 \times 10^{-2}]$	-1.006145	0.999976
Fibonacci ($\tau_k \propto F_k$)	24	$[1.535 \times 10^{-5}, 1.592 \times 10^{-2}]$	-1.002940	0.999988

Table 3: Log–log fits for ladder spectra in the asymptotic mid-band. Both constructions yield slopes near -1 with $R^2 \approx 1$, consistent with Proposition G.1.

I Limitations and next steps

From 1D rotation to many-body holography. The present paper uses the one-dimensional irrational rotation as a controlled prototype in which discrepancy and mismatch accumulation are explicit and computable. Extending $\sigma(x)$ to realistic many-body holographic models requires specifying (i) the microscopic scan and update architecture, (ii) the concrete measurement kernels $w_k^{(\varepsilon)}$, and (iii) the coarse-graining/scale-flow map relating window length N to spatial resolution.

Multi-window and spatial coarse-graining. Section 7 and Appendix G derive a quantitative $1/f$ band under an explicit ladder-aggregation closure (log-uniform relaxation times with approximately equal or bounded weights). Equal-weight aggregation yields an explicit prefactor, while bounded nonuniform weights preserve the $1/f$ mid-band and only modify the prefactor within a controlled range (Corollary G.2). The remaining open step is microscopic: to compute the ladder weights w_k from a concrete readout kernel family and an explicit scale flow, and to justify when the relaxation-mode superposition model is an accurate effective description of mismatch fluctuations.

Operational extraction of routing overhead. The lapse dictionary $\mathcal{N} = \kappa_0/\kappa$ is operationally meaningful only if proxies for compilation depth can be extracted (directly in explicit circuit models, or indirectly via observables such as phase delays and scattering times). Connecting these proxies to gravitational redshift in laboratory settings remains a key empirical interface.

Quantifying intelligence as a phase transition. The agent definition and survival criterion in Section 8 specify what should be computed (predictive gain vs. dissipation) but do not yet fix a unique microscopic model of control loops. Instantiating $\dot{F}_{\text{pred}}$ and $\dot{W}_{\text{diss}}$ in concrete architectures is the next step for turning the "intelligence phase" proposal into a quantitative theory.

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