Computational Action Principle II: Dynamical Einstein Gravity and Quantum Interfaces from Routing Overhead in HPA–Ω

Haobo Ma*

AELF PTE LTD.

#14-02, Marina Bay Financial Centre Tower 1, 8 Marina Blvd, Singapore 018981

December 28, 2025

Abstract

This manuscript develops a dynamical closure for the Computational Action Principle (CAP) in the unified Holographic Polar Arithmetic (HPA) and Ω-theory framework. Starting from auditable protocol objects—a physical interaction graph, a compilation/scheduling model, and a local task family—we define a routing-overhead field $\kappa(x, t)$ and an operational lapse $\mathcal{N}(x, t) = \kappa_0/\kappa(x, t)$ as a redshift factor. We then state a minimal set of additional closure assumptions under which κ can be identified with covariant effective fields and embedded into a local, second-order, diffeomorphism-invariant action. Within this universality class, the metric field equation is necessarily Einsteinian (Lovelock-type uniqueness), with discrepancy and implementation costs entering through an effective stress tensor. We further formulate a concrete scattering interface (Wigner–Smith time delay) enabling experimental or numerical calibration of κ via linewidth and delay measurements, and we outline reproducible in-repo numerics designed to test the weak-field and coarse-grained predictions with explicit error budgets.

Keywords: computational action principle; routing overhead; computational lapse; emergent gravity; Einstein equation; ADM split; POVM readout; Wigner–Smith time delay; Abel finite part; universality class.

Conventions. Unless otherwise stated, log denotes the natural logarithm and we set $c = 1$. We keep $\hbar$ explicit when discussing scattering time delay and calibration. We use the mostly-plus metric signature $(-, +, +, +)$. Protocol time is discrete: we write the tick index as $n \in \mathbb{Z}_{\geq 0}$ and convert to physical time by $t = n\tau_0$ when comparing to continuum coordinate time in the effective theory. Spatial coordinates are written as x in the continuum description and $v \in V$ on the underlying graph; coarse-grained continuum fields (e.g. $\kappa(x, t)$) are readout-scale representatives of the underlying discrete protocol observables.

Core symbols.

$\kappa(x, n)$	routing overhead (compilation depth) of the local task at tick n
κ_0	reference overhead used to define the computational lapse
τ_0	reference duration of one primitive tick (seconds per tick)
$\mathcal{N}$	computational lapse (redshift proxy), $\mathcal{N} = \kappa_0/\kappa$
N^i	ADM shift (gauge choice on slices)
h_{ij}	ADM spatial metric on constant-t slices
ρ	coarse-grained information density, $\rho = \chi^2$
χ	Fisher-amplitude scalar field
u	$u = \log(\kappa/\kappa_0) = -\log \mathcal{N}$ (so $u = \Phi$ in the weak-field dictionary)

*Email: auric@aelf.io

Contents

1 Introduction

This paper is a dynamical companion to the constructive and weak-field templates developed in the HPA–Ω program. The starting point is operational: finite observers access the world through finite-resolution readout, and local laws must be consistent with the compilation and scheduling constraints by which microscopic updates are physically realized. In this language a *routing overhead* κ quantifies the minimal depth required to realize a prescribed local update on a given hardware substrate. The corresponding *computational lapse* $\mathcal{N} = \kappa_0/\kappa$ is an auditable proxy for local redshift.

Scope and the role of assumptions. The main logical distinction in this manuscript is between (i) definitions and bounds that follow from the compilation model, and (ii) additional closure assumptions that embed those auditable quantities into a covariant effective theory. The latter step is unavoidable if one wants more than a static weak-field template: full dynamical gravity requires a prescription for how lapse, shift, and stress-energy are sourced by protocol data. Accordingly, we state an explicit *assumption ledger* and keep each bridge input testable, parameterized, and replaceable.
The closure assumptions used here are not presented as a derivation theorem from compilation depth alone. Rather, they specify an effective-theory universality class that is natural for finite observers on local substrates: locality reflects finite-range microscopic interactions and finite readout bandwidth; diffeomorphism invariance is asserted only for coarse-grained observables and is interpreted as protocol gauge (Section 4); and the restriction to second-order metric equations is the minimal ghost-free, well-posed choice within a low-energy EFT reading (Section 5). The content of CAP-II is therefore concentrated in the auditable lapse proxy and in the explicit source-sector closure, which can be falsified by quantitative fits and stability tests.

Contributions. The paper makes four concrete contributions:

1. We define a time-dependent routing-overhead field $\kappa(x, n)$ (or its coarse-grained representative $\kappa(x, t)$ with $t = n\tau_0$) and an operational local proper-time count $\tau_{\rm loc}(T; x)$ by cycle counting over a finite tick horizon T, and we state the redshift test targets in a finite-horizon form.

2. We formalize a minimal closure package that maps κ to covariant fields (an information density and Fisher-amplitude) and specifies a local, diffeomorphism-invariant, second-order effective action.

3. Under this package, we derive the Einstein equation with an explicit information stress tensor and summarize the ADM split, emphasizing where further constitutive identification is required for shift and momentum flow.

4. We give a scattering interface (Wigner–Smith time delay) and a reproducibility policy that allows numerical and experimental verification of lapse ratios and weak-field limits with controlled coarse-graining error.

Relation to existing manuscripts. CAP [1] provides the variational unification viewpoint and the static weak-field templates. HHU [2] develops constructive spacetime mechanisms and a computational-lapse gravity dictionary anchored in explicit compilation models. The present manuscript focuses on the dynamical bridge: how one passes from auditable routing overhead to a time-dependent covariant effective theory without conflating definitions with assumptions.

2 Assumption ledger and auditable objects

This manuscript separates three kinds of inputs:

1. *Auditable protocol objects*: hardware interaction data, a compilation/scheduling model, and a local task family. These determine $\kappa(x, n)$ (and hence, after readout smoothing, a coarse-grained $\kappa(x, t)$) and lapse ratios by definition and counting.

2. *Coarse-graining conventions*: how finite-resolution readout maps discrete protocol data to continuum fields at scale ε.

3. *Closure assumptions*: additional identifications required to embed the above into a covariant effective theory with dynamical gravity.

We now record the non-definitional assumptions as an explicit ledger.

Scales and units (recorded once). We distinguish four scales: (i) the microscopic spacing h of the hardware substrate (or the characteristic separation between physical sites), (ii) the readout scale ε of the observer kernel, (iii) a macroscopic variation scale L of coarse-grained fields, and (iv) the finite tick horizon T used in operational redshift tests. The intended continuum regime is

$$h \ll \varepsilon \ll L, \tag{1}$$

and tick costs are converted to physical time by a reference tick duration τ_0 (seconds per primitive tick).

Dimension conventions. With $c = 1$, we treat spacetime coordinates as having dimensions of length, so ∇_μ carries dimension L^{-1}. We take the Fisher-amplitude field χ to have dimension L^{-1} (equivalently $\rho = \chi^2$ has dimension L^{-2}), so that the scalar kinetic term in (19) has the same mass dimension as R without inserting an additional scale. Accordingly, λ_F is dimensionless in our normalization, ρ_0 carries dimension L^{-2} in Assumption 2.3, and $V(\chi^2)$ has dimension L^{-4}.

2.1 Structural assumptions (S)

Assumption 2.1 (Local compilation model)**.** *There exists a fixed physical interaction graph $G_{\text{phys}} = (V, E, w)$, where vertices are physical sites, edges are primitive two-site interactions, and $w : E \to \mathbb{R}_{>0}$ assigns tick costs. Primitive single-site operations are allowed at $O(1)$ tick cost. Parallel execution is permitted only on disjoint supports (matching constraints), with total tick cost given by the maximum occupied edge weight per layer.*
We treat $w(e)$ as a dimensionless tick count; the corresponding physical duration is $w(e)\tau_0$.

Assumption 2.2 (Local task family)**.** *For each continuum point x and protocol tick index n (equivalently, coarse-grained time $t = n\tau_0$), there is an associated bounded-diameter local update task $G_x(n)$ whose compilation depth under Assumption 2.1 defines $\kappa(x, n)$. The task family is stable under coarse-graining: after smoothing at scale ε the induced observables vary slowly on that scale.*

2.2 Identification and closure assumptions (C)

Assumption 2.3 (Cost-to-density identification)**.** *There exist constants $\rho_0 > 0$, $\kappa_0 > 0$ and an exponent $p > 0$ such that the coarse-grained information density obeys*

$$\rho(x, t) = \rho_0 \left(\frac{\kappa(x, t)}{\kappa_0}\right)^p \qquad \textit{in the effective domain of interest.} \tag{2}$$

The exponent p and calibration ρ_0 are treated as fit parameters, not as derived constants.
We assume $\kappa(x, t)$ is strictly positive and bounded on the domain of interest, so that $\mathcal{N} = \kappa_0/\kappa$ is well-defined and bounded away from 0.

Remark 2.4 (Micro-motivation and universality test for the cost-to-density map)**.** *Assumption 2.3 is a closure input: it maps an auditable scheduling depth into an effective scalar density used in the covariant source sector. The assumption is motivated by a generic mechanism in constrained parallel execution on local graphs: schedule length is controlled by* congestion *(how much weighted work must traverse/occupy a local bottleneck) and by* dilation *(the typical distance over which a task must route dependencies). In many routing/scheduling settings, one has bounds of the form*

$$\text{Depth} \gtrsim \text{Congestion}, \tag{3}$$

and constructive upper bounds scaling like $\text{Depth} \lesssim C\,(\text{Congestion} + \text{Dilation})$ *up to polylogarithmic factors, for appropriate notions of congestion/dilation and admissible schedules; see, e.g., [3, 4].*

Operationally, this means that when the local task family at readout scale ε is parameterized by a coarse-grained load *observable (expected weighted number of required primitive interactions per unit volume per cycle), the measured routing overhead κ is a monotone proxy for that load in the effective domain. In the absence of additional local scales after coarse-graining, a power-law ansatz in κ/κ_0 is the simplest closed form compatible with multiplicative rescalings, and p becomes a universality exponent capturing how load renormalizes into the effective density used in the action.*

Stability criterion (falsifiable). *Fix a calibration rule for κ_0 and a readout scale ε. Across multiple task families and hardware realizations that are (ε, δ)-equivalent at the observable level (Definition 4.2), estimate p by a log–log regression of ρ against κ/κ_0 over the same effective domain. If the inferred p is stable (within uncertainty) under changes of task family, address protocol, and moderate changes of ε respecting $h \ll \varepsilon \ll L$, then Assumption 2.3 identifies a genuine universality class; systematic drift signals that a richer constitutive map $\rho = \rho(\kappa, \nabla\kappa, \ldots)$ is required.*

Assumption 2.5 (Fisher-amplitude field)**.** *Define the Fisher-amplitude scalar field $\chi(x, t)$ by $\chi = \sqrt{\rho}$. The effective action depends on ρ only through χ and its covariant derivatives, up to a local potential term $V(\chi^2)$.*

Assumption 2.6 (Covariant effective action)**.** *There exists a local, diffeomorphism-invariant effective action $S[g, \chi, \psi_m]$ in four dimensions such that the metric field equation is of second differential order and the matter sector couples covariantly.*

Assumption 2.7 (Protocol gauge and momentum density identification)**.** *In the ADM formulation we regard the shift N^i as a gauge choice (spatial coordinates on each slice). In the* minimal *CAP-II closure used for quantitative targets, we work in a protocol-rest gauge with $N^i = 0$ on the domain of interest. The momentum density j_i entering the momentum constraint is identified with the standard slice quantity $j_i := -T_{\mu\nu} n^\mu h^\nu{}_i$ built from the total stress tensor derived from the effective action. Under the cost-to-density identification, the information-sector contribution is therefore determined by κ via $\chi = \sqrt{\rho_0}(\kappa/\kappa_0)^{p/2}$.*

2.3 Readout and regularization assumptions (R)

Assumption 2.8 (Kernel readout regularity)**.** *Finite-resolution readout at scale ε is modeled by a positive kernel K_ε inducing coarse-grained observables by convolution/smoothed averaging. The kernel has finite effective bandwidth and sufficient regularity so that discrete difference operators approximate continuum derivatives with controlled error when the lattice spacing $h \ll \varepsilon$.*

A standard concrete model is a mollifier family $K_\varepsilon(x) = \varepsilon^{-d} K(x/\varepsilon)$ with $K \geq 0$, $\int_{\mathbb{R}^d} K = 1$, and K smooth with sufficient decay/compact support.

Assumption 2.9 (Canonical regularization path)**.** *When a regulated-to-continuum limit requires extracting a finite constant term, we fix a canonical prescription (Abel regularization along $r \uparrow 1$*

and finite-part extraction). Different finite parts are interpreted as different local counterterm choices in the effective action.

Testability. Each assumption above is associated with a measurable consequence: lapse ratios from tick counting; parameter identification in the weak-field limit; and linewidth/time-delay calibration in the scattering interface. Sections 3–9 make these targets explicit.

3 From protocol to fields: $\kappa(x, t)$ and computational lapse

3.1 Hardware substrate, scheduling, and compilation depth

Fix the physical interaction graph $G_{\text{phys}} = (V, E, w)$ of Assumption 2.1. We interpret each edge $e \in E$ as admitting a primitive two-site gate at cost $w(e)$ ticks, and each vertex as admitting primitive single-site gates at $O(1)$ cost. A schedule is a sequence of parallel layers, where each layer is a set of primitive gates with disjoint supports. The cost of a layer is the maximum tick cost among the gates executed in that layer; the cost of a schedule is the sum of layer costs.

Protocol realizations are defined on discrete sites $v \in V$ and tick indices $n \in \mathbb{Z}_{\geq 0}$. To compare with the continuum effective theory we work with coarse-grained representatives at readout scale ε; for readability we typically suppress ε and write (x, t) with $t = n\tau_0$ for the corresponding physical time (Appendix E records representative bandwidth-error estimates).

Definition 3.1 (Compilation depth and routing overhead)**.** *Let $G_x(n)$ be the local update task at point x and tick index n (Assumption 2.2). Define the compilation depth* $\text{Depth}(G_x(n); G_{\text{phys}})$ *as the minimum schedule cost required to realize $G_x(n)$ using the primitive operations of G_{phys}. Define the routing overhead field*

$$\kappa(x, n) \;:=\; \text{Depth}(G_x(n); G_{\text{phys}}). \tag{4}$$

3.2 Local cycle counting and operational proper time

Fix a reference tick duration τ_0 (seconds per primitive tick) and a reference overhead κ_0 in a chosen calibration region (e.g. a far-field region for asymptotically flat benchmark fits, where one may set $\mathcal{N} \approx 1$ by convention). Consider a long protocol horizon of T ticks. At a fixed x, the maximal number of completed local tasks by time T is

$$C_x(T) \;:=\; \left\lfloor \frac{T}{\kappa(x)} \right\rfloor, \tag{5}$$

where we suppress the explicit n-dependence for a quasi-static background over the horizon. We define the operational local proper time by

$$\tau_{\text{loc}}(T; x) \;:=\; C_x(T)\,\tau_0. \tag{6}$$

Proposition 3.2 (Finite-horizon proper-time scaling)**.** *Assume $\kappa(x, n)$ is constant in n over the horizon, written as $\kappa(x)$. Then*

$$\left| \tau_{\text{loc}}(T; x) - \frac{T\,\tau_0}{\kappa(x)} \right| \leq \tau_0, \tag{7}$$

and therefore

$$\frac{\tau_{\text{loc}}(T; x)}{T} = \frac{\tau_0}{\kappa(x)} + O\!\left(\frac{1}{T}\right). \tag{8}$$

In particular, for two points x_1, x_2 with constant overheads over $[0, T]$ one has the finite-horizon redshift ratio

$$\frac{\tau_{\text{loc}}(T; x_1)}{\tau_{\text{loc}}(T; x_2)} = \frac{\kappa(x_2)}{\kappa(x_1)} + O\!\left(\frac{1}{T}\right). \tag{9}$$

Remark 3.3 (Time-dependent overhead)**.** *When $\kappa(x,n)$ varies with the tick index n, a natural cycle count over a T-tick horizon is the maximal number of completed tasks*

$$C_x(T) \;:=\; \max\Big\{m \in \mathbb{Z}_{\geq 0} : \sum_{j=0}^{m-1} \kappa(x,j) \leq T\Big\}, \tag{10}$$

with $\tau_{\mathrm{loc}}(T;x) := C_x(T)\tau_0$ as before. If $\kappa_{\min} \leq \kappa(x,j) \leq \kappa_{\max}$ holds throughout the horizon, then the crude but auditable bounds

$$\left\lfloor \frac{T}{\kappa_{\max}} \right\rfloor \leq C_x(T) \leq \left\lfloor \frac{T}{\kappa_{\min}} \right\rfloor \tag{11}$$

follow immediately. The present paper uses the quasi-static regime for clean test targets; fully dynamical closure is addressed through the covariant action and the ADM formulation in Sections 5–7.

3.3 Computational lapse and a weak-field dictionary

Definition 3.4 (Computational lapse)**.** *Define the computational lapse field by*

$$\mathcal{N}(x,n) \;:=\; \frac{\kappa_0}{\kappa(x,n)}. \tag{12}$$

When using the continuum effective description we write $\mathcal{N}(x,t)$ for the corresponding coarse-grained representative with $t = n\tau_0$.

The quantity $\mathcal{N}$ is an auditable, protocol-level object: it is defined by compilation depth and can be inferred by cycle counting. To compare with a relativistic effective description we adopt the standard weak-field dictionary, working in a protocol-rest slicing in which the shift is negligible (or set to zero; Section 7). More generally, in ADM variables one has $g_{00} = -\mathcal{N}^2 + h_{ij}N^iN^j$, so the identification below is the shiftless limit.

$$g_{00}(x,t) \;\approx\; -\mathcal{N}(x,t)^2, \qquad \Phi(x,t) \;:=\; -\log \mathcal{N}(x,t), \tag{13}$$

so that Φ plays the role of a Newtonian-type potential in the static regime. Section 10 records the consistency targets in the Newtonian limit.

4 Continuum limit and universality class

This section records the minimal conditions under which protocol-level differences (addressing family, local compilation details, tie-breaking rules) do not obstruct an effective continuum description at finite readout resolution. The guiding principle is operational: invariance is asserted only for *coarse-grained observables* accessible to a finite observer.

4.1 Coarse-graining by kernel readout

Fix a readout scale $\varepsilon > 0$. Assumption 2.8 models readout by a positive kernel K_ε of bandwidth $\sim \varepsilon^{-1}$. For a field-like observable F we define the coarse-grained quantity

$$F_\varepsilon(x) \;:=\; \int_{\mathbb{R}^d} K_\varepsilon(x-y)\, F(y)\, \mathrm{d}y, \tag{14}$$

or the analogous discrete convolution on a lattice. We emphasize that F_ε is the object with operational meaning; F itself is a model-dependent microscopic representative.

Remark 4.1 (Derivative scaling under mollification)**.** *For a mollifier family $K_\varepsilon(x) = \varepsilon^{-d}K(x/\varepsilon)$ with K smooth, one has the standard scaling*

$$\|\partial^\alpha F_\varepsilon\|_{L^\infty(\Omega)} \le \|F\|_{L^\infty(\Omega_\varepsilon)}\ \|\partial^\alpha K\|_{L^1(\mathbb{R}^d)}\ \varepsilon^{-|\alpha|}, \tag{15}$$

where Ω_ε is an ε-neighborhood of Ω. Thus, at fixed readout scale, only a bounded number of derivatives are operationally meaningful, and their natural magnitude scales like $\varepsilon^{-|\alpha|}$. See, e.g., [5] for standard mollification estimates.

4.2 Protocol gauge and (ε, δ)-equivalence

Definition 4.2 ((ε, δ)-equivalent protocol realizations)**.** *Two protocol realizations are called (ε, δ)-equivalent on a domain Ω if their coarse-grained lapse fields satisfy*

$$\sup_{x\in\Omega} |\mathcal{N}_{1,\varepsilon}(x) - \mathcal{N}_{2,\varepsilon}(x)|\ \le\ \delta, \tag{16}$$

and their coarse-grained source observables (when used) differ by at most δ in the same norm.

The definition is intentionally weak: it regards different microscopic representatives as the same physical state whenever they cannot be distinguished at the readout scale. In particular, the choice of address family (Hilbert vs Morton/Z-order, etc.) is interpreted as a *protocol gauge* when it changes only sub-ε structure.

Proposition 4.3 (Universality of band-limited observables)**.** *Assume two realizations are (ε, δ)-equivalent on Ω. Let $\mathcal{O}_\varepsilon$ be any observable that depends on the lapse only through a bounded number of derivatives at scale ε (equivalently, a band-limited functional in the readout bandwidth). Then there exists a constant $C_{\mathcal{O}}$ (depending on the readout kernel and on $\mathcal{O}_\varepsilon$ but not on microscopic representatives) such that*

$$\sup_{x\in\Omega} |\mathcal{O}_\varepsilon[\mathcal{N}_1](x) - \mathcal{O}_\varepsilon[\mathcal{N}_2](x)|\ \le\ C_{\mathcal{O}}\,\delta. \tag{17}$$

Corollary 4.4 (Derivative stability at fixed bandwidth)**.** *Assume* $\sup_{x\in\Omega} |\mathcal{N}_1(x) - \mathcal{N}_2(x)| \le \delta$ *and K_ε is a mollifier as in Assumption 2.8. Then for any multi-index α there exists a constant C_α (depending only on K and α) such that*

$$\sup_{x\in\Omega} |\partial^\alpha \mathcal{N}_{1,\varepsilon}(x) - \partial^\alpha \mathcal{N}_{2,\varepsilon}(x)| \le C_\alpha\,\delta\,\varepsilon^{-|\alpha|}. \tag{18}$$

This makes explicit how stability degrades as one asks for higher derivatives at fixed readout scale.

Interpretation. Proposition 4.3 is the operative form of diffeomorphism invariance used in this program: invariance is stated not for microscopic encodings, but for the coarse-grained observables that define the effective continuum physics.

Clarification (microscopic gauge vs. operational invariance). Microscopic choices such as address family can affect the representative $\mathcal{N}$ at scales below ε. However, the effective field theory is formulated for $\mathcal{N}_\varepsilon$ and its band-limited functionals, and it is only at this level that invariance is asserted and tested.

5 Effective action and closure principle

To move from an auditable lapse proxy to a dynamical spacetime theory, one must specify how protocol observables embed into covariant fields and how those fields enter an effective action. This step is a *closure*: it cannot be obtained from compilation depth alone, and it is therefore stated explicitly as assumptions in Section 2.

5.1 Action ansatz

Under Assumptions 2.3–2.6 we adopt an effective action of the form

$$S[g,\chi,\psi_m] = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi G} - \lambda_F\, g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - V(\chi^2) + \mathcal{L}_m(g,\psi_m)\right]. \tag{19}$$

Here R is the Ricci scalar, Λ is a cosmological term, χ is the Fisher-amplitude field, and $\mathcal{L}_m$ denotes additional effective matter degrees of freedom. The potential $V(\chi^2)$ encodes mismatch penalties and model-dependent implementation costs not captured by the quadratic Fisher term.

Remark 5.1 (Relation to Fisher information)**.** *Writing $\rho = \chi^2$, one has*

$$g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi = \frac{1}{4}\,\frac{g^{\mu\nu}\nabla_\mu\rho\,\nabla_\nu\rho}{\rho}. \tag{20}$$

Thus the scalar kinetic term is a covariant Fisher-information penalty on the coarse-grained density profile, a structure familiar from information-theoretic variational principles; see, e.g., [6, 7].

Remark 5.2 (Sign choice and ghost-freedom)**.** *For the scalar sector to be ghost-free in the effective field theory, the kinetic coefficient should satisfy $\lambda_F > 0$ (with our mostly-plus convention). We adopt this sign throughout and treat λ_F as a nonnegative fit parameter.*

Remark 5.3 (Boundary terms and a well-posed metric variation)**.** *If the spacetime region has a boundary, the Einstein–Hilbert term requires a boundary contribution for a well-posed Dirichlet variational principle. One standard choice is the Gibbons–Hawking–York term [8, 9]*

$$S_{\mathrm{GHY}} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} K\,\sqrt{|h|}\,\mathrm{d}^3x, \tag{21}$$

where h is the induced metric and K is the trace of the extrinsic curvature of the boundary. In this manuscript we either assume boundary conditions under which the boundary term does not contribute, or we implicitly include S_{GHY} in the gravitational sector.

5.2 Why the metric equation is Einstein

The left-hand side of the metric field equation is fixed *within the chosen closure class* by the structural requirements of locality, diffeomorphism invariance, and second-order metric equations (Assumption 2.6). In four dimensions, Lovelock-type uniqueness implies that the only symmetric, divergence-free rank-2 tensor built from the metric and its derivatives up to second order is $G_{\mu\nu} + \Lambda g_{\mu\nu}$ up to an overall coupling. Therefore the micro-to-macro content of CAP lies in the *source sector* determined by χ and $\mathcal{L}_m$, not in modifying the geometric skeleton. See [10] for the original uniqueness statement and standard GR texts for the effective-field-theory reading.

Remark 5.4 (Higher-derivative corrections in an EFT reading)**.** *In a generic low-energy effective field theory of gravity, the action includes higher-curvature operators (e.g. R^2, $R_{\mu\nu}R^{\mu\nu}$) suppressed by a cutoff scale. CAP-II's* minimal *closure restricts to a second-order metric equation as an explicit modeling choice (Assumption 2.6); higher-derivative terms can be incorporated systematically as controlled corrections once a readout/cutoff scale is specified. See, e.g., [11, 12].*

5.3 Regularization, finite parts, and counterterms

Assumption 2.9 fixes a canonical rule for extracting finite constants from regulated sums or traces (Abel regularization and finite-part extraction). At the effective-action level, different finite parts correspond to different choices of local counterterms and hence to different renormalization conditions. The role of the ledger is to keep this scheme choice explicit and to isolate it from the purely operational definition of κ and $\mathcal{N}$.

6 Field equations from variation

We record the dynamical equations implied by the action (19). Full variational details are deferred to Appendix B.

6.1 Metric variation and the information stress tensor

Theorem 6.1 (Einstein equation with information stress)**.** *Varying* (19) *with respect to the metric yields*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \left(T^{(m)}_{\mu\nu} + T^{\text{info}}_{\mu\nu} \right), \tag{22}$$

where $T^{(m)}_{\mu\nu}$ *is the matter stress tensor and*

$$T^{\text{info}}_{\mu\nu} = 2\lambda_F \left(\nabla_\mu \chi \, \nabla_\nu \chi - \frac{1}{2} g_{\mu\nu} (\nabla\chi)^2 \right) - g_{\mu\nu} V(\chi^2). \tag{23}$$

6.2 Amplitude equation and conservation

Proposition 6.2 (Fisher-amplitude equation)**.** *Varying* (19) *with respect to* χ *yields*

$$2\lambda_F \, \Box\chi - \frac{\mathrm{d}V}{\mathrm{d}\chi} = 0. \tag{24}$$

By diffeomorphism invariance one has $\nabla^\mu G_{\mu\nu} = 0$, hence the total stress tensor is covariantly conserved:

$$\nabla^\mu \left(T^{(m)}_{\mu\nu} + T^{\text{info}}_{\mu\nu} \right) = 0. \tag{25}$$

When (24) holds and the matter sector is covariantly conserved (or exchanges flow through explicit couplings in $\mathcal{L}_m$), the split into matter and information sectors is consistent.

6.3 κ-formulation under the cost-to-density map

The effective dynamics becomes explicitly closed in terms of the auditable routing overhead once one fixes the cost-to-density identification (Assumption 2.3) and a potential V. Write

$$u(x,t) := \log\!\left(\frac{\kappa(x,t)}{\kappa_0} \right), \qquad \rho = \rho_0 \, \mathrm{e}^{pu}, \qquad \chi = \sqrt{\rho_0} \, \mathrm{e}^{\frac{p}{2}u}. \tag{26}$$

Proposition 6.3 (Closed scalar equation for $u = \log(\kappa/\kappa_0)$)**.** *Assume Assumptions 2.3–2.5. Then the Fisher-amplitude equation* (24) *is equivalent to*

$$\lambda_F \, p \, \Box u + \lambda_F \, \frac{p^2}{2} \, (\nabla u)^2 = \frac{1}{\chi} \frac{\mathrm{d}V}{\mathrm{d}\chi}, \qquad \chi = \sqrt{\rho_0} \, \mathrm{e}^{\frac{p}{2}u}, \tag{27}$$

where $(\nabla u)^2 := g^{\mu\nu} \nabla_\mu u \nabla_\nu u$.

Proof. This is a direct chain-rule substitution for $\chi = \sqrt{\rho_0} \, \mathrm{e}^{\frac{p}{2}u}$; see Appendix B.3. □

Proposition 6.4 (Potential-derivative reconstruction from u)**.** *Assume V depends only on $\rho = \chi^2$ (Assumption 2.5), so $V(\chi^2) = \widehat{V}(\rho)$. Then* (27) *is equivalently*

$$\widehat{V}'(\rho) = \frac{1}{2} \left(\lambda_F \, p \, \Box u + \lambda_F \, \frac{p^2}{2} \, (\nabla u)^2 \right), \qquad \rho = \rho_0 \, \mathrm{e}^{pu}, \tag{28}$$

where $\widehat{V}'(\rho) = \mathrm{d}\widehat{V}/\mathrm{d}\rho$. In particular, given a solution (g, u) one can reconstruct $\widehat{V}'$ along the image of ρ.

Remark 6.5 (Normalization of $\widehat{V}$)**.** *The additive constant in $\widehat{V}$ is not physically observable at the level of equations of motion: it shifts the effective cosmological term by a constant. Accordingly, one may impose a normalization condition such as $\widehat{V}(\rho_0) = 0$ without loss of generality, absorbing any constant offset into Λ.*

Proposition 6.6 (Information stress in terms of u)**.** *With the same identifications, the information stress tensor* (23) *can be written as*

$$T^{\text{info}}_{\mu\nu} = \lambda_F \frac{p^2}{2} \chi^2 \left(\nabla_\mu u \, \nabla_\nu u - \frac{1}{2} g_{\mu\nu} (\nabla u)^2 \right) - g_{\mu\nu} V(\chi^2), \qquad \chi = \sqrt{\rho_0} \, \mathrm{e}^{\frac{p}{2} u}. \tag{29}$$

Quantitative closure and fitting viewpoint. Equations (22) and (27) form a closed covariant system for $(g_{\mu\nu}, u)$ once $(p, \rho_0, \lambda_F, V)$ and the matter sector are specified. Operationally, κ is measured (by compilation logs or via the scattering proxy in Section 9), hence u is data. CAP-II uses this to impose a self-consistency fit: parameters are chosen so that the solution's lapse (in a chosen gauge) matches the operational lapse $\mathcal{N} = \kappa_0/\kappa$ after coarse-graining.

6.4 Minimal potential families and long-range behavior

To make the closure predictive, one must specify a family for the local potential $V(\chi^2)$. A minimal requirement is *stability* of the reference background $u = 0$ (equivalently $\kappa = \kappa_0$), which suggests that V has a local minimum at $\chi^2 = \rho_0$. One convenient parametrization is to write $V(\chi^2) = \widehat{V}(\rho)$ with $\rho = \chi^2$ and expand around ρ_0:

$$\widehat{V}(\rho) = \widehat{V}(\rho_0) + \frac{m_\rho^2}{2} (\rho - \rho_0)^2 + O\Big((\rho - \rho_0)^3\Big), \tag{30}$$

where m_ρ sets an effective stiffness scale.

Remark 6.7 (Massless versus massive regime)**.** *In the linearized regime $|u| \ll 1$ one has $\rho - \rho_0 \approx \rho_0 p \, u$. If $m_\rho > 0$, the scalar sector has an effective correlation length (Compton scale) and deviations sourced through u are suppressed beyond that scale. If instead the quadratic term vanishes (effectively massless), then static spherically symmetric solutions generically carry scalar hair and differ from Schwarzschild; see the classical Fisher/JNW family for Einstein gravity coupled to a massless scalar [13, 14]. CAP-II treats the Schwarzschild lapse as a benchmark fit target in a regime where scalar backreaction is either negligible or short-ranged compared to the exterior window; deviations can be used to constrain the potential family.*

7 ADM split and dynamical closure

The variational equations of Section 6 are covariant. To make contact with protocol time (ticks) and with dynamical initial-value formulations, we summarize the standard $3+1$ (ADM) decomposition. The key point for CAP-II is not the ADM algebra itself, but where additional constitutive closure is required to interpret lapse and shift in terms of protocol observables.
In the ADM equations below, t denotes the continuum coordinate time of the effective theory; comparison to protocol logs uses $t = n\tau_0$ after coarse-graining (Section 1 and the conventions in the front matter).

7.1 Kinematics: lapse, shift, and extrinsic curvature

Write the spacetime metric in ADM form:

$$\mathrm{d}s^2 = -\mathcal{N}^2 \, \mathrm{d}t^2 + h_{ij} \left(\mathrm{d}x^i + N^i \mathrm{d}t \right) \left(\mathrm{d}x^j + N^j \mathrm{d}t \right), \tag{31}$$

where $\mathcal{N}$ is the lapse, N^i is the shift, and h_{ij} is the induced spatial metric on constant-t slices. The extrinsic curvature is

$$K_{ij} := \frac{1}{2\mathcal{N}} \left(\dot{h}_{ij} - D_i N_j - D_j N_i \right), \tag{32}$$

where D_i is the covariant derivative of h_{ij} and a dot denotes ∂_t.

7.2 Constraints vs evolution

The Einstein equation splits into constraint equations on each slice and evolution equations for (h_{ij}, K_{ij}). Schematically, the Hamiltonian constraint and momentum constraint take the form

$$^{(3)}R + K^2 - K_{ij}K^{ij} = 16\pi G\, \rho_{\rm tot} + 2\Lambda, \tag{33}$$

$$D_j \left(K^{ij} - h^{ij} K \right) = 8\pi G\, j^i_{\rm tot}, \tag{34}$$

where $^{(3)}R$ is the scalar curvature of h_{ij}, and $\rho_{\rm tot}, j^i_{\rm tot}$ are the total energy density and momentum flux seen by the slice normal.

To display the dynamical content explicitly, the evolution equation for the spatial metric is

$$\dot{h}_{ij} = -2\mathcal{N} K_{ij} + D_i N_j + D_j N_i, \tag{35}$$

while the evolution of the extrinsic curvature takes the schematic form

$$\begin{aligned} \dot{K}_{ij} = &-D_i D_j \mathcal{N} + \mathcal{N} \left({}^{(3)}R_{ij} + K K_{ij} - 2K_{ik} K^k{}_j \right) + \mathcal{L}_{\vec{N}} K_{ij} \\ &- 8\pi G \mathcal{N} \left(S_{ij} - \tfrac{1}{2} h_{ij} (S - \rho_{\rm tot}) \right) + \Lambda \mathcal{N}\, h_{ij}, \end{aligned} \tag{36}$$

where $^{(3)}R_{ij}$ is the Ricci tensor of h_{ij}, $\mathcal{L}_{\vec{N}}$ denotes the Lie derivative along the shift vector, and $S_{ij} := h_i{}^\mu h_j{}^\nu T_{\mu\nu}$ with $S := h^{ij} S_{ij}$. See standard ADM references for the full system and conventions [15, 16].

Closure point. The protocol-level definition of κ directly provides an auditable lapse proxy $\mathcal{N} = \kappa_0/\kappa$ (Section 3). To obtain a quantitative initial-value system one must also specify a spatial coordinate gauge (shift) and identify the slice densities $(\rho_{\rm tot}, j^i_{\rm tot}, S_{ij})$. In GR the shift is pure gauge, and in the minimal CAP-II closure we adopt a protocol-rest gauge $N^i = 0$ (Assumption 2.7). The remaining quantities are then determined by the effective stress tensor obtained from the action (Section 6), with κ entering through the $\kappa \mapsto \chi$ identification. More elaborate realizations may use protocol flow to define a preferred coordinate choice, but the minimal fit targets in Section 10.2 do not require this.

8 Quantum interfaces: readout, unitary evolution, and bandwidth

The effective gravitational equations of CAP-II live at the coarse-grained level. Quantum mechanics enters as an interface theory for finite observers: the observer has an effective Hilbert space, applies local unitaries (scan/update), and accesses outcomes through finite-resolution readout instruments. This section records the minimum interface structure used later, emphasizing what is assumed and what is merely a change of representation.

8.1 Effective observer sector and POVM readout

Let $\mathcal{H}_{\rm eff}$ be an effective observer sector and let ρ be a density operator on $\mathcal{H}_{\rm eff}$. Finite-resolution readout at scale ε is modeled by a POVM $\{E^{(\varepsilon)}_k\}_k$ with $\sum_k E^{(\varepsilon)}_k = \mathbf{1}$, giving Born probabilities

$$P^{(\varepsilon)}_k = \mathrm{Tr}(\rho\, E^{(\varepsilon)}_k). \tag{37}$$

The corresponding instrument (state update rule) may be written in Kraus form $\rho \mapsto M_k \rho M_k^\dagger / P_k$ with $E_k = M_k^\dagger M_k$. Any such POVM can be implemented by a dilation (system–ancilla unitary followed by a projective measurement on the ancilla). This is a standard consequence of Naimark and Stinespring representation theorems; see, e.g., [17–20].

8.2 Bandwidth and controlled continuum operators

Assumption 2.8 asserts that readout is sufficiently regular and band-limited so that discrete difference operators approximate continuum derivatives on coarse-grained observables. Operationally, this is the condition under which a continuum effective field theory is meaningful: the observer cannot resolve sub-ε microstructure, and the induced operators act on F_ε with controllable error when $h \ll \varepsilon$. Appendix E records a representative error estimate for gradients/Laplacians under band-limited kernel readout.

8.3 Unitary evolution and representation choices

Whenever a one-parameter unitary family U_t implements an automorphism of the effective algebra by $A \mapsto U_t^\dagger A U_t$, one may represent the same expectation data either by evolving observables (Heisenberg) or by evolving the state representative (Schrödinger). CAP-II does not claim to derive this unitary structure from Einstein gravity; rather, it treats it as part of the observer-interface layer and uses it to define measurable quantities (e.g. scattering delay) that can be calibrated against the operational lapse.

9 Scattering apparatus and Wigner–Smith time delay

This section states a minimal scattering interface that turns an operational delay measurement into a calibrated proxy for routing overhead. The goal is to provide a concrete experimental and numerical verification route for lapse ratios without requiring direct access to the underlying compilation log.

9.1 Scattering apparatus (minimal Hamiltonian model)

Fix an internal (finite) region with Hamiltonian H coupled to M asymptotic channels (leads). In a standard effective description, the coupling is encoded by a matrix W mapping channel states into the internal region, leading to an effective non-Hermitian Hamiltonian

$$H_{\text{eff}}(E) := H - \mathrm{i}\pi\, W W^\dagger, \tag{38}$$

and an on-shell scattering matrix of the form [21, 22]

$$S(E) = \mathbf{1} - 2\pi\mathrm{i}\, W^\dagger \left(E - H_{\text{eff}}(E)\right)^{-1} W. \tag{39}$$

This model covers, at the level of interfaces, a broad class of platforms: mesoscopic transport, microwave networks, photonic structures, and circuit-QED scattering measurements.

9.2 Wigner–Smith operator and time delay

Consider an M-channel scattering apparatus with on-shell scattering matrix $S(E) \in U(M)$. The Wigner–Smith time-delay operator [23, 24] is

$$Q(E) \;:=\; -\mathrm{i}\,\hbar\, S(E)^\dagger \, \frac{\mathrm{d}S}{\mathrm{d}E}(E), \tag{40}$$

and the mean Wigner–Smith delay is

$$\tau_{\text{WS}}(E) \;:=\; \frac{1}{M}\mathrm{Tr}Q(E). \tag{41}$$

9.3 Resonance linewidth calibration

Near an isolated resonance at energy E_0 with linewidth γ, the scattering matrix admits a Breit–Wigner-type parametrization. In the single-resonance regime, the peak time delay scales as

$$\tau_{\mathrm{WS}}(E_0) \;\approx\; \frac{4\hbar}{\gamma}, \tag{42}$$

up to apparatus-dependent prefactors that can be calibrated by a reference region.
For a single channel ($M = 1$) one may write $S(E) = \mathrm{e}^{2\mathrm{i}\delta(E)}$. In the Breit–Wigner approximation the phase shift takes the standard form

$$\delta(E) \;\approx\; \arctan\!\left(\frac{\gamma/2}{E_0 - E}\right), \tag{43}$$

see, e.g., [25]. so that $\tau_{\mathrm{WS}}(E) = 2\hbar\,\mathrm{d}\delta/\mathrm{d}E$ and $\tau_{\mathrm{WS}}(E_0) = 4\hbar/\gamma$, consistent with (42).

Remark 9.1 (Channel conventions)**.** *The prefactor in* (42) *depends on the convention used for* τ_{WS} *(mean over channels, partial delay, or a particular incoming state) and on how* γ *is defined from the pole structure. The present paper fixes the convention by* (40)–(41) *and uses* (42) *as a benchmark in the single-resonance regime [23–25].*

9.4 From delay to routing overhead

Define the dimensionless delay proxy

$$\kappa_{\mathrm{WS}}(E) \;:=\; \frac{\tau_{\mathrm{WS}}(E)}{\tau_0}, \tag{44}$$

where τ_0 is the reference tick duration. The interface assumption is that a localized resonance probe at location x has a characteristic resonance energy $E_0(x)$ and linewidth $\gamma(x)$ such that

$$\kappa(x) \;\approx\; \kappa_{\mathrm{WS}}(E_0(x)) \approx \; \frac{4\hbar}{\gamma(x)\tau_0}. \tag{45}$$

This identification makes lapse ratios directly testable:

$$\frac{\mathcal{N}(x_1)}{\mathcal{N}(x_2)} = \frac{\kappa(x_2)}{\kappa(x_1)} \approx \frac{\gamma(x_1)}{\gamma(x_2)}. \tag{46}$$

In practice one calibrates the overall scale by a reference region x_0: define $\gamma_0 := \gamma(x_0)$ and $\kappa_0 := 4\hbar/(\gamma_0\tau_0)$ in the same single-resonance convention, so that $\kappa(x)/\kappa_0 \approx \gamma_0/\gamma(x)$ within the calibrated bandwidth window. Appendix G records standard trace identities, basis-invariance properties, and loss/dispersion considerations relevant for turning measured S-data into calibrated κ ratios. Section 10 records consistency targets, and Appendix C comments on how probe localization interacts with coarse-graining.

10 Benchmark limits

This section summarizes the consistency targets CAP-II is required to reproduce in appropriate limits. The emphasis is not on claiming uniqueness, but on recording the regimes in which the routing-overhead lapse should match known weak-field and cosmological templates under the closure assumptions.

10.1 Newtonian / weak-field limit

In the static weak-field regime, write the metric in scalar-potential form

$$\mathrm{d}s^2 = -(1+2\phi)\,\mathrm{d}t^2 + (1-2\phi)\,\mathrm{d}\mathbf{x}^2, \qquad |\phi| \ll 1. \tag{47}$$

Then $g_{00} \approx -(1+2\phi)$ and the lapse satisfies $\mathcal{N} \approx 1+\phi$. We denote the ADM lapse by $\mathcal{N}$ (often written N in the GR literature). With the dictionary $\Phi = -\log \mathcal{N}$, one has $\Phi \approx -\phi$ to leading order. The 00 Einstein equation reduces to the Poisson equation

$$\Delta\phi = 4\pi G\,\rho_{\mathrm{tot}}, \tag{48}$$

so in vacuum $\Delta\phi = 0$ and asymptotic flatness forces $\phi(r) = -GM/r$ for an isolated source. See, e.g., [26, 27].

In terms of the auditable routing overhead, the computational-lapse dictionary gives $u := \log(\kappa/\kappa_0) = -\log\mathcal{N} =: \Phi$. Therefore, in the same weak-field gauge one has $\Phi \approx -\phi$ and hence

$$\Delta\Phi \;=\; -4\pi G\,\rho_{\mathrm{tot}} \qquad (|\phi| \ll 1). \tag{49}$$

10.2 Direct fit target: Schwarzschild lapse from operational κ

For a static, spherically symmetric exterior region in GR one has the Schwarzschild lapse

$$\mathcal{N}_{\mathrm{Schw}}(r) = \sqrt{1 - \frac{2GM}{r}}, \qquad r > 2GM. \tag{50}$$

In the weak-field regime $r \gg 2GM$ this becomes

$$\mathcal{N}_{\mathrm{Schw}}(r) = 1 - \frac{GM}{r} + O\!\left(\frac{1}{r^2}\right), \qquad \frac{\kappa(r)}{\kappa_0} = \frac{1}{\mathcal{N}(r)} = 1 + \frac{GM}{r} + O\!\left(\frac{1}{r^2}\right), \tag{51}$$

where the last identity uses the operational lapse definition $\mathcal{N} = \kappa_0/\kappa$. Therefore a minimal quantitative check is a linear fit of $\kappa(r)/\kappa_0 - 1$ against $1/r$ in an exterior window where weak-field and finite-resolution conditions hold.

Exact inversion (Schwarzschild benchmark). If one assumes the exact Schwarzschild form $\mathcal{N}^2(r) = 1 - 2GM/r$ in an exterior window, then the benchmark admits a pointwise inversion:

$$GM = \frac{r}{2}\left(1 - \mathcal{N}(r)^2\right) = \frac{r}{2}\left(1 - \frac{1}{(\kappa(r)/\kappa_0)^2}\right) = \frac{r}{2}\left(1 - \left(\frac{\gamma(r)}{\gamma_0}\right)^2\right), \tag{52}$$

where the last identity uses the linewidth proxy $\gamma/\gamma_0 \approx \mathcal{N}$. The weak-field regressions in Section 10.3 are the leading-order linearizations of this inversion.

Remark 10.1 (Scalar hair and why Schwarzschild is a benchmark, not an identity)**.** *CAP-II couples gravity to an additional scalar sector determined by the cost-to-density identification. In Einstein gravity with a* massless *scalar, the generic static spherically symmetric solution is not Schwarzschild but the Fisher/JNW family [13, 14]. Accordingly, the Schwarzschild lapse is used here as a benchmark fit target in a controlled regime: either the scalar sector is effectively short-ranged (pinned by the potential) or its backreaction is negligible in the chosen exterior window. Departures from the benchmark provide quantitative constraints on the scalar potential family and coupling scale.*

Using the scattering proxy. If $\kappa(r)$ is inferred from Wigner–Smith delay via (45), then in the same regime one predicts

$$\frac{\gamma(r)}{\gamma_0} = \frac{\kappa_0}{\kappa(r)} = \mathcal{N}(r) = 1 - \frac{GM}{r} + O\!\left(\frac{1}{r^2}\right), \tag{53}$$

so GM can be estimated by a weighted least-squares fit of $1 - \gamma(r)/\gamma_0$ against $1/r$. The same fit yields a falsifiable consistency check: the inferred GM must be stable under changing the coarse-graining scale ε within the regime $h \ll \varepsilon \ll r$.

10.3 Quantitative regression target and uncertainty (minimal)

Let $(r_i, \kappa_i)_{i=1}^n$ be measurements (or compiled estimates) of routing overhead at radii r_i in an exterior window, and define

$$x_i := \frac{1}{r_i}, \qquad y_i := \frac{\kappa_i}{\kappa_0} - 1. \tag{54}$$

In the weak-field model (51) one has the linear regression target

$$y_i = (GM)\, x_i + \epsilon_i, \tag{55}$$

where ϵ_i collects finite-resolution bias (controlled by ε), finite-horizon tick-count rounding error, and measurement noise.

Remark 10.2 (Calibration offset and intercept fit)**.** *Model* (55) *fits through the origin and is appropriate when κ_0 (or γ_0) is calibrated in a region where the benchmark normalization $\mathcal{N} \approx 1$ applies. If instead the reference is taken in a finite region, the weak-field fit should include an intercept nuisance parameter b:*

$$y_i = (GM)\, x_i + b + \epsilon_i. \tag{56}$$

Given nonnegative weights w_i (e.g. inverse variances), the weighted least-squares estimator is

$$\widehat{GM} = \frac{\sum_{i=1}^n w_i x_i y_i}{\sum_{i=1}^n w_i x_i^2}. \tag{57}$$

If the errors are independent with $\mathrm{Var}(\epsilon_i) = \sigma_i^2$ and $w_i = \sigma_i^{-2}$, then the standard estimated variance is

$$\mathrm{Var}(\widehat{GM}) \approx \left(\sum_{i=1}^n w_i x_i^2 \right)^{-1}, \tag{58}$$

up to model-misspecification bias.

Window conditions (what makes the fit meaningful). The fit window should satisfy simultaneously:

- *Weak field:* $r_i \gg 2GM$ so that truncation at $O(1/r^2)$ is controlled.
- *Readout separation:* $\varepsilon \ll r_i$ so that coarse-graining does not wash out the radial profile.
- *Microscopic separation:* $h \ll \varepsilon$ so that continuum derivatives on coarse-grained fields are meaningful.

The stability condition across ε is therefore not cosmetic but a key falsifiable test: if $\widehat{GM}$ drifts systematically as ε varies within the separation regime, the closure is inconsistent with a Schwarzschild-like exterior interpretation.

Equivalent regression from linewidth ratios. When κ is inferred through the Wigner–Smith linewidth proxy (45), one may instead set

$$y_i := 1 - \frac{\gamma_i}{\gamma_0}, \tag{59}$$

and use the same estimator (57) under the weak-field prediction (53).

10.4 Homogeneous cosmology with lapse

For an FLRW ansatz with homogeneous lapse,

$$\mathrm{d}s^2 = -\mathcal{N}^2(t)\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\Sigma_k^2, \tag{60}$$

proper time is $\mathrm{d}\tau = \mathcal{N}(t)\,\mathrm{d}t$ and the Friedmann equations take their standard form in τ. This reduction provides a controlled setting for testing how the cost-to-density map (Assumption 2.3) and the potential $V(\chi^2)$ impact effective expansion histories.

10.5 Linearized sector (optional target)

If the effective theory is to capture dynamical gravitational degrees of freedom beyond the Newtonian sector, the linearized equations around a background should reproduce standard wave-like propagation at long wavelengths. In strictly local microscopic substrates (QCA/PQCA), causality bounds (Lieb–Robinson-type) constrain signal/front velocities; the effective description must remain consistent with such bounds in its domain of validity.

11 Reproducible numerics (in-repo)

This repository follows an in-repo reproducibility discipline: numerical checks are generated by scripts committed alongside the manuscript, and the manuscript records the exact commands, parameters, and random seeds required to reproduce tables and figures. This section states the reproducibility contract for CAP-II.

11.1 Build instructions

The paper is compiled from the project directory by:

```
latexmk-pdf-interaction=nonstopmode-halt-on-errormain.tex
```

11.2 Determinism and randomness

All numerical scripts used by this manuscript must:

- take parameters only via explicit command-line flags (with defaults printed on start), and
- fix all pseudo-randomness by a recorded integer seed.

When randomness is not required, scripts should be deterministic by construction.

11.3 Generated artifacts

To keep the LaTeX build deterministic and lightweight, scripts write LaTeX fragments (typically table rows) under a dedicated directory (e.g. `sections/generated/`) which are then included by the paper. The build remains valid even if generated artifacts are absent; in that case the manuscript should fall back to static tables or omit the corresponding figure.

11.4 Reference scripts included in this repository

Finite-horizon rounding error for τ_{loc}. This script generates rows for Table 1 and empirically illustrates the bound in Proposition 3.2 (with τ_0 scaled out):

```
python3scripts/exp_tau_loc_floor_error.py--outsections/generated/tau_loc_floor_error_
rows.tex
```

T	κ	$\lfloor T/\kappa \rfloor$	T/κ	$\lvert \lfloor T/\kappa \rfloor - T/\kappa \rvert$
10	3	3	3.33333	0.333333
10	7	1	1.42857	0.428571
10	11	0	0.909091	0.909091
10	37	0	0.27027	0.27027
25	3	8	8.33333	0.333333
25	7	3	3.57143	0.571429
25	11	2	2.27273	0.272727
25	37	0	0.675676	0.675676
100	3	33	33.3333	0.333333
100	7	14	14.2857	0.285714
100	11	9	9.09091	0.0909091
100	37	2	2.7027	0.702703
250	3	83	83.3333	0.333333
250	7	35	35.7143	0.714286
250	11	22	22.7273	0.727273
250	37	6	6.75676	0.756757
1000	3	333	333.333	0.333333
1000	7	142	142.857	0.857143
1000	11	90	90.9091	0.909091
1000	37	27	27.027	0.027027

Table 1: **Finite-horizon rounding error in cycle counting.** The last column is always < 1, consistent with (7) after scaling out τ_0.

Breit–Wigner peak Wigner–Smith delay calibration. This script generates rows for Table 2 and numerically verifies the peak scaling $\tau_{\mathrm{WS}}(E_0) \approx 4\hbar/\gamma$ using a central-difference derivative:

```
python3scripts/exp_ws_breit_wigner_numeric_check.py--outsections/generated/ws_breit_
wigner_rows.tex--hbar1.0--dE1e-6
```

Weak-field Schwarzschild regression (data-driven). Given a CSV file containing radii r_i and either the ratio κ_i/κ_0 or γ_i/γ_0, the following script computes the weighted least-squares estimate (57) and writes a one-row summary:
This repository ships small baseline CSVs under `data/` generated from the exact Schwarzschild lapse with $GM = 1$ (in the same length units as r) to validate the pipeline; differences between the two linearized modes are therefore expected at $O(1/r^2)$ due to truncation.

```
python3scripts/fit_schw_weakfield_wls.py--modekappa_ratio--in_csvdata/schw_kappa_
ratio.csv--outsections/generated/schw_weakfield_fit_kappa_rows.tex
```

```
python3scripts/fit_schw_weakfield_wls.py--modegamma_ratio--in_csvdata/schw_gamma_
ratio.csv--outsections/generated/schw_weakfield_fit_gamma_rows.tex
```

Synthetic regression with an explicit error budget (sanity check). To demonstrate recovery of GM within stated uncertainties under a simple noise model, the following script generates synthetic weak-field data for $y = (GM)/r$ with additive Gaussian noise of standard deviation σ_y, performs the origin-constrained WLS fit, and reports the estimated standard error $\mathrm{SE}(\widehat{GM})$ and a z-score $(\widehat{GM} - GM)/\mathrm{SE}$:

γ	ΔE	$\tau_{WS}(E_0)$ (num.)	$4\hbar/\gamma$	rel. error
0.2	1.00e-06	20	20	1.000e-10
0.5	1.00e-06	8	8	1.600e-11
1	1.00e-06	4	4	4.000e-12
2	1.00e-06	2	2	1.000e-12
5	1.00e-06	0.8	0.8	1.600e-13

Table 2: **Wigner–Smith peak delay for a Breit–Wigner resonance (single channel).** The numerical delay uses $\hbar = 1$ and a finite-difference derivative step ΔE.

mode	n	$\widehat{GM}$	RMSE
kappa_ratio	12	1.02321	7.597e-05
gamma_ratio	12	1.00765	2.487e-05

Table 3: **Weak-field Schwarzschild benchmark fit.** The script fits the slope through the origin in the model $y = (GM)/r$ using either $y = \kappa/\kappa_0 - 1$ or $y = 1 - \gamma/\gamma_0$.

```
python3scripts/exp_schw_weakfield_synth_wls_demo.py--GM1.0--n25--r_min50--r_
max800--sigma_y1e-4--seed0--outsections/generated/schw_weakfield_synth_wls_row.tex
```

End-to-end microscopic example: interaction graph → $\kappa(x)$ → coarse-grained lapse → WS recovery. This script provides an explicit microscopic triple (G_{phys}, G_x, scheduler): *(i)* a weighted interaction graph G_{phys} given by `data/demo_chain_edges.csv`, *(ii)* a local clock-task family G_x parameterized by nodes at radii in `data/demo_chain_nodes.csv` (each task requires interaction with a fixed reference node), and *(iii)* a shortest-path scheduler in which the compilation depth is the weighted path cost plus a unit local cost. The induced lapse profile $N(x) = \kappa_0/\kappa(x)$ is compared against the corresponding normalized Schwarzschild target and against a WS-based recovery via linewidth calibration with stated relative error.

```
python3scripts/demo_microscopic_chain_to_ws.py--nodes_csvdata/demo_chain_nodes.
csv--edges_csvdata/demo_chain_edges.csv--out_rowssections/generated/demo_microscopic_
chain_rows.tex--out_metricssections/generated/demo_microscopic_chain_metrics.tex--GM1.
0--eps_r50--ws_modelinewidth--sigma_rel_gamma1e-3--seed0
```

12 Discussion and open problems

CAP-II is designed to make the dynamical bridge explicit: compilation-level objects define lapse ratios operationally, while covariant dynamics requires additional closure inputs. This separation makes the theory falsifiable in layers: some predictions are definition-level and must hold in any realization of the compilation model; others are closure-dependent and can be tested to discriminate between constitutive choices.

12.1 Assumption sensitivity

Three sensitivities are central:

n	$r_{\min}$	$r_{\max}$	σ_y	GM	$\widehat{GM}$	SE	RMSE	z
25	50	800	1.0e-04	1	0.998663	3.45e-03	1.09e-04	-0.39

Table 4: **Synthetic weak-field regression with uncertainty.** This table is a pipeline sanity check: under the stated noise model, the estimator (57) recovers GM within the predicted uncertainty budget.

r	N_{target}	N_{ε}	N_{WS}	rel. err(N_{ε})	rel. err(N_{WS})
800	1.00000000	0.99999986	1.00000000	1.40e-07	0.00e+00
600	0.99958220	0.99958206	0.99724708	1.41e-07	2.34e-03
400	0.99874608	0.99863216	0.99712821	1.14e-04	1.62e-03
300	0.99790926	0.99760842	0.99733978	3.01e-04	5.71e-04
250	0.99723930	0.99690281	0.99528848	3.37e-04	1.96e-03
200	0.99623351	0.99530642	0.99522444	9.31e-04	1.01e-03
150	0.99455493	0.99247182	0.99379728	2.09e-03	7.62e-04
120	0.99287352	0.99056293	0.99111500	2.33e-03	1.77e-03
100	0.99118926	0.98933048	0.98896043	1.88e-03	2.25e-03
80	0.98865748	0.98817795	0.98791883	4.85e-04	7.47e-04
60	0.98442338	0.98713698	0.98447406	2.76e-03	5.15e-05
50	0.98102294	0.98666486	0.97946586	5.75e-03	1.59e-03

Table 5: **Explicit microscopic end-to-end lapse test.** N_{target} is the normalized Schwarzschild lapse at the radii in `data/demo_chain_nodes.csv`. N_{ε} is the Gaussian coarse-grained lapse induced by the graph-computed $\kappa(x)$ (with bandwidth parameter ε in r-units). N_{WS} is recovered by a calibrated WS linewidth proxy with a stated relative linewidth error $\sigma_{\text{rel},\gamma}$.

- *Cost-to-density map.* The exponent p in Assumption 2.3 and the potential V control how routing overhead backreacts as effective stress-energy; Remark 2.4 records a micro-motivated congestion viewpoint and a stability criterion for treating p as a universality exponent.
- *Gauge and slicing choices.* In the minimal closure we fix a protocol-rest shift gauge (Assumption 2.7); beyond this, physically preferred coordinate choices may be induced by protocol flow, and should be treated as part of the model specification when comparing time-dependent data.
- *Readout bandwidth.* Assumption 2.8 controls which microscopic differences are unobservable and therefore which invariances can be claimed at the effective level.

12.2 Relation to scalar–tensor frameworks and solar-system constraints

The minimal CAP-II closure (19) is Einstein gravity with an additional scalar source sector, not a Brans–Dicke-type modification of the geometric coupling. Accordingly, the leading post-Newtonian coefficients of the metric are Einsteinian in regimes where the exterior is well-approximated by a Schwarzschild-like template. Observable departures arise when the information scalar is light enough to carry exterior hair (Fisher/JNW-type behavior) or when its stress-energy produces measurable higher-order corrections in a benchmark window. Appendix F records the corresponding PPN positioning, the standard light-deflection/Shapiro-delay observ-

n	ε	WS mode	$\sigma_{\mathrm{rel},\gamma}$	RMSE(N_ε)	RMSE(N_{WS})	max rel. err(N_{WS})
12	50	linewidth	1.00e-03	2.114e-03	1.438e-03	2.336e-03

Table 6: **End-to-end error summary.** The coarse-graining width ε is given in the same length units as r. In linewidth mode the WS recovery uses $\kappa_{\mathrm{WS}}(E_0) \approx 4\hbar/(\gamma\tau_0)$ with a synthetic relative linewidth uncertainty $\sigma_{\mathrm{rel},\gamma}$.

ables, and the massive-vs-light scalar regimes in terms of $(p, \rho_0, \lambda_F, V)$ through the constitutive identification.

12.3 Relation to trace/regularization mechanisms

Abel-type regularization and finite-part extraction provide a canonical way to assign constants to regulated orbit sums and traces. In a dynamical setting, the same scheme choice appears as a renormalization condition in the effective action (Section 5.3), and therefore must be treated as part of the model specification rather than as an after-the-fact numerical trick.

12.4 Open problems

One open problem is to derive, from microscopic protocol data, a physically preferred coordinate gauge or current interpretation in genuinely time-dependent regimes, beyond the minimal protocol-rest gauge used for the benchmark fits. A second open problem is to integrate the dynamical closure with spectral/trace-formula mechanisms in a single unified framework without conflating the observer-interface layer with the covariant field layer.

13 Conclusion

We presented CAP-II as an explicit dynamical closure program for computational-lapse gravity. At the protocol level, routing overhead $\kappa(x, t)$ and cycle counting define lapse ratios operationally. To obtain a dynamical spacetime theory one must add a controlled package of closure assumptions mapping κ to covariant fields and specifying a local, diffeomorphism-invariant, second-order effective action. Under these assumptions the macroscopic metric equation is Einsteinian with an information stress tensor, and the remaining dynamical freedom is isolated in the constitutive identification of shift and currents.

We also formulated a concrete scattering interface (Wigner–Smith time delay) that allows empirical calibration of κ and direct tests of lapse ratios via linewidth ratios. Future work will focus on deriving the shift/current closure from microscopic protocol flow and on expanding the reproducible numerical suite for genuinely time-dependent benchmarks.

A Appendices

B Variation details

This appendix records the core variational identities behind Section 6. We emphasize that the purpose is auditability: all assumptions are isolated in the action ansatz (19), while the derivation is standard.

B.1 Metric variation

For a covariant action $S = \int \sqrt{-g}\,\mathcal{L}$, variation of the Einstein–Hilbert term yields

$$\delta\left(\sqrt{-g}\,R\right) = \sqrt{-g}\,G_{\mu\nu}\,\delta g^{\mu\nu} + (\text{boundary term}). \tag{61}$$

For the Fisher-amplitude term one uses $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\,g_{\mu\nu}\delta g^{\mu\nu}$ and the definition $(\nabla\chi)^2 = g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi$ to obtain the stress tensor (23).

B.2 Scalar variation

Varying with respect to χ gives the Euler–Lagrange equation

$$2\lambda_F\,\Box\chi - \frac{\mathrm{d}V}{\mathrm{d}\chi} = 0, \tag{62}$$

where $\Box = \nabla^\mu\nabla_\mu$. When the scalar equation holds, the divergence of $T^{\text{info}}_{\mu\nu}$ reduces to a term proportional to the scalar equation, yielding the consistency statement in (25).

B.3 $u = \log(\kappa/\kappa_0)$ chain rule identity

This subsection records the elementary chain rule behind Proposition 6.3. Under Assumption 2.3, write $u = \log(\kappa/\kappa_0)$ so that

$$\chi = \sqrt{\rho} = \sqrt{\rho_0}\,\mathrm{e}^{\frac{p}{2}u}. \tag{63}$$

Then

$$\nabla_\mu\chi = \frac{p}{2}\,\chi\,\nabla_\mu u, \qquad \Box\chi = \nabla^\mu\nabla_\mu\chi = \frac{p}{2}\,\chi\,\Box u + \frac{p^2}{4}\,\chi\,(\nabla u)^2, \tag{64}$$

where $(\nabla u)^2 = g^{\mu\nu}\nabla_\mu u\nabla_\nu u$. Substituting into the scalar equation $2\lambda_F\Box\chi - \mathrm{d}V/\mathrm{d}\chi = 0$ and dividing by $\chi > 0$ yields

$$\lambda_F\,p\,\Box u + \lambda_F\,\frac{p^2}{2}\,(\nabla u)^2 = \frac{1}{\chi}\,\frac{\mathrm{d}V}{\mathrm{d}\chi}, \tag{65}$$

which is (27).

C ADM details and closure interfaces

This appendix complements Section 7 by recording standard ADM identities and highlighting where CAP-specific closure enters.

C.1 Constraint quantities

Let n^μ be the future-directed unit normal to a spatial slice. Define the energy density and momentum flux by

$$\rho_{\text{tot}} := T_{\mu\nu}n^\mu n^\nu, \qquad j_i^{\text{tot}} := -T_{\mu\nu}n^\mu h^\nu{}_i. \tag{66}$$

These are the quantities appearing in (33)–(34).

C.2 Gauge versus constitutive content

In pure GR, lapse and shift encode gauge freedom in foliating spacetime. In CAP-II, the lapse is also an operational observable (Section 3.3). This dual role forces an explicit separation between:

- a gauge choice used to set up an initial-value formulation, and
- a constitutive identification that maps protocol data to the slice quantities $(\rho_{\text{tot}}, j_i^{\text{tot}})$.

Assumption 2.7 asserts that such a constitutive map exists and is local within the readout bandwidth.

C.3 Preferred shift from protocol flow (optional extension)

The minimal CAP-II closure used for benchmark fits works in a protocol-rest gauge with $N^i = 0$ (Assumption 2.7). In genuinely time-dependent regimes it can be advantageous to use protocol data to define a *preferred* spatial frame, i.e. a preferred shift beyond pure gauge fixing.
A concrete route is to treat directed protocol activity as a measurable coarse-grained current. For example, compilation logs can provide, at readout scale ε, oriented edge-usage counts (net executed two-site primitives across an oriented cut per tick) which define an effective spatial flow field $v^i(x,t)$ after coarse-graining. Given such a v^i, one may impose a *comoving* slicing condition

$$N^i(x,t) = v^i(x,t), \tag{67}$$

or, equivalently, select coordinates in which the measured protocol flow has vanishing spatial components. This turns part of the gauge freedom into constitutive content and is therefore an additional modeling assumption, to be recorded alongside the ledger when used.
Operationally, adopting a preferred shift requires at least:

- an auditable definition of a coarse-grained protocol current (directed edge utilization, task-dependency flow, or a related transport observable), and
- a compatibility rule relating that current to the slice momentum density j^i_{tot} in (34) (e.g. identifying a frame in which j^i_{tot} is minimized or vanishes for the information sector).

The present manuscript isolates this issue by keeping the benchmark targets in a shiftless gauge; preferred-shift closure is left as an explicit extension point for time-dependent datasets.

D Regularization notes: finite parts and counterterms

When orbit sums or traces diverge in regulated-to-continuum passages, CAP adopts a canonical finite-part prescription. The present manuscript uses this only at the level of model specification: the choice of finite part is part of the effective action via renormalization conditions.

D.1 Abel regularization and finite part

Given a divergent series $\sum_{n\geq 0} a_n$, define its Abel generating function

$$A(r) := \sum_{n\geq 0} a_n r^n, \qquad 0 < r < 1. \tag{68}$$

If $A(r)$ admits an asymptotic expansion as $r \uparrow 1$ of the form

$$A(r) = \frac{c_{-m}}{(1-r)^m} + \cdots + \frac{c_{-1}}{1-r} + c_0 + o(1), \tag{69}$$

we define the Abel finite part by FP $\sum_{n\geq 0} a_n := c_0$.

D.2 Scheme dependence

Different prescriptions for extracting c_0 correspond, at the level of an effective action, to different local counterterm choices. Therefore CAP-II treats the regularization scheme as part of the model specification (Assumption 2.9) and isolates it from the operational definition of κ.

D.3 A concrete counterterm correspondence (one worked interface)

This subsection records a minimal calculation showing how a finite-part choice translates into a local counterterm in the CAP-II effective action.

Constant finite parts shift the cosmological term. Suppose a regulated protocol-derived scalar contribution to the effective Lagrangian density takes the form

$$\Delta\mathcal{L}_{\mathrm{reg}}(r) = A(r), \qquad 0 < r < 1, \tag{70}$$

with an Abel expansion as $r \uparrow 1$:

$$A(r) = \frac{c_{-1}}{1-r} + c_0 + o(1). \tag{71}$$

Under the Abel finite-part rule, the renormalized contribution is $\Delta\mathcal{L}_{\mathrm{ren}} := c_0$. If instead one chooses a different finite part $c_0 \mapsto c_0 + \delta c_0$, then the action shifts by

$$\delta S = \int \mathrm{d}^4x \sqrt{-g}\, \delta c_0, \tag{72}$$

which is equivalent to a shift of the cosmological term in (19):

$$\Lambda \;\mapsto\; \Lambda - 8\pi G\, \delta c_0. \tag{73}$$

Thus, at the level of local covariant dynamics, scheme dependence of constant finite parts is exactly the usual cosmological-constant counterterm freedom.

Density-proportional finite parts shift the potential. Similarly, if a regulated contribution takes the form

$$\Delta\mathcal{L}_{\mathrm{reg}}(r) = B(r)\, \chi^2, \qquad B(r) = \frac{b_{-1}}{1-r} + b_0 + o(1), \tag{74}$$

then a finite-part change $b_0 \mapsto b_0 + \delta b_0$ shifts the action by

$$\delta S = \int \mathrm{d}^4x \sqrt{-g}\, (\delta b_0)\, \chi^2, \tag{75}$$

which is equivalent to adding a local counterterm to the scalar potential:

$$V(\chi^2) \;\mapsto\; V(\chi^2) - (\delta b_0)\, \chi^2. \tag{76}$$

More generally, finite-part choices for protocol-level traces can renormalize the coefficients of any local invariant allowed in the chosen effective-theory class; CAP-II fixes a canonical prescription (Assumption 2.9) and interprets any residual finite ambiguity as an explicit renormalization condition on (Λ, V).

E Readout bandwidth and controlled differential operators

This appendix records a representative controlled-approximation statement for differential operators under kernel readout. The goal is not to optimize constants, but to exhibit the scale separation structure $h \ll \varepsilon$.

E.1 A model estimate (with standard references)

Let f be a smooth function on a domain $\Omega \subset \mathbb{R}^d$. Let f_h denote its sampling on a lattice of spacing h, and let K_ε be a smooth kernel of bandwidth $\sim \varepsilon^{-1}$. Define the coarse-grained field $f_{\varepsilon,h} := K_\varepsilon * f_h$. Then discrete symmetric differences acting on $f_{\varepsilon,h}$ approximate continuum derivatives with an error controlled by the ratio h/ε:

$$\|\nabla_h f_{\varepsilon,h} - \nabla f_\varepsilon\|_{L^\infty(\Omega)} \;\lesssim\; C \left(\frac{h}{\varepsilon}\right)^2, \tag{77}$$

under standard smoothness assumptions on f and K_ε. An analogous estimate holds for discrete Laplacians.

Such statements are standard in numerical analysis (finite differences) combined with mollification/smoothing estimates; see, e.g., [5, 28].

Interpretation. Assumption 2.8 asserts that the observer's readout bandwidth enforces precisely this kind of scale separation, ensuring that continuum operators are meaningful on coarse-grained observables even when the microscopic substrate is discrete and protocol-dependent.

E.2 Microscopic "protocol gauge" perturbations and ε/h suppression (model bound)

The universality notion of Section 4.2 treats microscopic representation choices (e.g. address order or tie-breaking rules) as a protocol gauge whenever they affect only sub-ε structure. Here we record one representative bound exhibiting explicit ε/h suppression for a class of lattice-scale perturbations.

For simplicity we state the estimate on a regular lattice; the same proof extends to bounded-degree graphs with h interpreted as the microscopic spacing and with degree-dependent constants.

Proposition E.1 (Divergence-form perturbations are suppressed by h/ε)**.** *Let κ_h and κ'_h be two lattice fields on a spacing-h lattice in $\mathbb{R}^d$. Assume their difference admits a discrete divergence representation*

$$\delta\kappa_h := \kappa'_h - \kappa_h = \nabla_h \cdot J_h, \tag{78}$$

where J_h is an edge/flux field supported on edges and bounded in sup norm by $\|J_h\|_\infty \le J_{\max}$. Let K_ε be a smooth mollifier as in Assumption 2.8 and set $\delta\kappa_{\varepsilon,h} := K_\varepsilon * \delta\kappa_h$. Then there exists a constant C_K depending only on the kernel family such that

$$\|\delta\kappa_{\varepsilon,h}\|_{L^\infty} \;\le\; C_K \, J_{\max} \, \frac{h}{\varepsilon}. \tag{79}$$

Sketch of proof. Write $\delta\kappa_{\varepsilon,h} = K_\varepsilon * (\nabla_h \cdot J_h)$ and apply discrete integration by parts to move ∇_h onto the smooth kernel. One obtains $\delta\kappa_{\varepsilon,h} = -(\nabla_h K_\varepsilon) * J_h$. Since $\|\nabla_h K_\varepsilon\|_{L^1} \lesssim (h/\varepsilon)\|\nabla K\|_{L^1}$ for a mollifier family, the bound (79) follows.

On a bounded-degree graph of maximum degree Δ, the same argument yields an additional factor polynomial in Δ through the discrete divergence definition. If the microscopic protocol gauge choice acts only within a bounded task diameter D (Assumption 2.2), then $J_{\max}$ can be taken to scale at most polynomially in (Δ, D) for that task family, making the suppression explicit.

F Post-Newtonian and solar-system constraints (Einstein gravity with an information scalar)

This appendix positions the CAP-II source sector relative to standard scalar–tensor frameworks and records the minimal post-Newtonian (PPN) implications needed for solar-system tests. The goal is not to re-derive the full PPN formalism, but to make explicit what is fixed by the Einsteinian geometric skeleton and what must be constrained by the information-sector constitutive inputs.

F.1 Relation to scalar–tensor theories

The minimal CAP-II closure uses the action (19), i.e. Einstein–Hilbert gravity with an additional scalar field χ and covariant matter $\mathcal{L}_m(g, \psi_m)$. In particular, in this manuscript:

- the gravitational coupling G is constant in the action, and
- matter couples minimally to the metric (no direct χ–matter coupling is assumed in $\mathcal{L}_m$).

This differs from Brans–Dicke and more general scalar–tensor theories, where the scalar typically modifies the effective gravitational coupling (Jordan-frame $F(\varphi)R$ terms) and/or couples nontrivially to matter in the Einstein frame; see, e.g., [29–31].
Consequently, CAP-II in its minimal form is a metric theory with Einstein field equations and an additional stress tensor contribution. Deviations from Schwarzschild in exterior regions arise not from a modified left-hand side, but from nontrivial information-sector stress-energy $T^{\text{info}}_{\mu\nu}$ (and from any additional matter sector used in a given realization).

F.2 PPN parameters in the minimal closure

In standard PPN gauge (isotropic spatial coordinates), the weak-field metric of a static source is written schematically as

$$g_{00} = -1 + 2U - 2\beta U^2 + O(U^3), \tag{80}$$

$$g_{ij} = \Big(1 + 2\gamma U + O(U^2)\Big)\,\delta_{ij}, \tag{81}$$

where $U = GM/r$ is the Newtonian potential and (γ, β) are theory parameters; see, e.g., [30,31]. For Einstein gravity with constant G and minimally coupled matter, one has

$$\gamma = 1, \qquad \beta = 1, \tag{82}$$

independent of the detailed constitution of the source. In CAP-II this statement applies to the *geometric skeleton*: the post-Newtonian coefficients are those of GR in the regime where the exterior metric is well-approximated by a Schwarzschild-like solution.
Empirically, solar-system tracking constrains γ at the level $|\gamma - 1| \ll 1$; see [31] for current bounds and a consolidated discussion.

A check in a non-vacuum exterior (massless scalar/JNW family). To make the above concrete in a setting where the exterior is not vacuum, consider Einstein gravity coupled to a free massless scalar (the $V \equiv 0$ limit of (19) with $\lambda_F > 0$). The generic static spherically symmetric solution is the Fisher/JNW family [13, 14], which can be written in curvature coordinates as

$$\mathrm{d}s^2 = -\left(1 - \frac{b}{r}\right)^{\nu} \mathrm{d}t^2 + \left(1 - \frac{b}{r}\right)^{-\nu} \mathrm{d}r^2 + \left(1 - \frac{b}{r}\right)^{1-\nu} r^2 \mathrm{d}\Omega^2, \qquad 0 < \nu \le 1, \tag{83}$$

with $\nu = 1$ recovering Schwarzschild. Transforming to an isotropic radius ρ and expanding at large ρ yields

$$g_{00} = -1 + \frac{\nu b}{\rho} - \frac{\nu^2 b^2}{2\rho^2} + O(\rho^{-3}), \tag{84}$$

$$g_{ij} = \left(1 + \frac{\nu b}{\rho} + O(\rho^{-2})\right)\delta_{ij}, \tag{85}$$

so identifying $GM = \nu b/2$ one reads off $\gamma = \beta = 1$ at post-Newtonian order. Thus, even when the exterior contains scalar stress-energy, the leading PPN coefficients remain Einsteinian; constraints arise from higher-order terms and from departures from a Schwarzschild benchmark in a chosen exterior window.

F.3 Standard solar-system observables (leading order)

At leading post-Newtonian order, the classic tests depend only on γ and β. In particular, for light deflection at impact parameter b and Shapiro time delay for a radar signal with endpoints at radii r_1, r_2, one has (restoring c for clarity)

$$\delta\theta_{\text{light}} = \frac{1+\gamma}{2}\,\frac{4GM}{bc^2} = \frac{4GM}{bc^2} \qquad (\gamma = 1), \tag{86}$$

$$\Delta t_{\text{Shapiro}} \approx \frac{1+\gamma}{2}\,\frac{4GM}{c^3}\log\left(\frac{4r_1r_2}{b^2}\right) = \frac{2GM}{c^3}\log\left(\frac{4r_1r_2}{b^2}\right) \qquad (\gamma = 1), \tag{87}$$

see [30–32] for conventions and refinements.
In CAP-II's operational dictionary (Section 3.3), the weak-field potential is $\Phi = -\log\mathcal{N} = \log(\kappa/\kappa_0)$ and $g_{00} \approx -\mathcal{N}^2$ in a shiftless gauge. Therefore, once a dataset provides $\kappa(x)$ (by compilation logs or by the WS interface) and a benchmark exterior interpretation identifies GM in an exterior window (Section 10.2), the standard leading-order solar-system observables follow in that same window.

F.4 Constraints on the information scalar: massive vs. light regimes

The parameter dependence specific to CAP-II enters through the source sector $T^{\text{info}}_{\mu\nu}$ and the $\kappa \mapsto \chi$ constitutive map. Solar-system consistency is therefore most naturally phrased as a constraint that the information sector does not generate observable departures from the benchmark exterior templates in the relevant window.

Massive (pinned) regime. Assume V has a local minimum at the background $\chi = \chi_0$ with $V'(\chi_0) = 0$. Linearizing (24) gives

$$\Box\,\delta\chi - m_\chi^2\,\delta\chi = 0, \qquad m_\chi^2 := \frac{V''(\chi_0)}{2\lambda_F}, \tag{88}$$

so $\delta\chi$ is Yukawa-suppressed beyond the Compton length $\ell_\chi := m_\chi^{-1}$. In this regime, an exterior window with length scales $r \gg \ell_\chi$ is effectively Schwarzschild-like up to exponentially small corrections from the scalar gradients, and the benchmark fits of Section 10.2 are self-consistent. A conservative solar-system consistency condition is that the scalar be short-ranged compared to the smallest impact parameters b used in classic time-delay/deflection tests, i.e.

$$m_\chi\, b \gg 1 \qquad (\text{or equivalently } \ell_\chi \ll b). \tag{89}$$

For example, taking b of order a solar radius provides a concrete benchmark scale. In terms of the ρ-expansion family (30) with $V(\chi^2) = \widehat{V}(\rho)$ and $\rho = \chi^2$, one has near the minimum $\rho_0 = \chi_0^2$ that $V''(\chi_0) = 4\chi_0^2\,\widehat{V}''(\rho_0)$, so

$$m_\chi^2 = \frac{2\chi_0^2}{\lambda_F}\,\widehat{V}''(\rho_0). \tag{90}$$

Thus a lower bound on $\widehat{V}''(\rho_0)$ (given λ_F and χ_0) is a directly interpretable short-range condition. In the explicit quadratic family (30), $\widehat{V}''(\rho_0) = m_\rho^2$, hence

$$m_\chi^2 = \frac{2\rho_0}{\lambda_F}\,m_\rho^2, \qquad (\rho_0 = \chi_0^2). \tag{91}$$

The exponent p enters these constraints through the constitutive identification $\chi = \sqrt{\rho_0}(\kappa/\kappa_0)^{p/2}$: in the weak-field regime $u = \log(\kappa/\kappa_0)$ is small and $\delta\chi/\chi_0 \approx (p/2)\,u$. Thus, for a fixed operational lapse profile $u(x)$, smaller p suppresses the amplitude of scalar-sector excursions and therefore the size of $T^{\text{info}}_{\mu\nu}$ corrections in an exterior window.

Light (nearly massless) regime and JNW deviations. If m_χ is very small on solar-system scales, static spherically symmetric exteriors generically carry scalar hair and are described by Fisher/JNW-type families (or their potential-deformed analogues). Although $\gamma = \beta = 1$ at leading PPN order (Section F.2), the exact exterior metric can differ from Schwarzschild at higher orders and in the near field. In CAP-II this regime is treated as an *observable deviation channel*: departures from the Schwarzschild benchmark constrain the potential family $V(\chi^2)$ and the effective coupling scale set by (p, ρ_0, λ_F) through the $\kappa \mapsto \chi$ identification.

F.5 Fifth-force interpretation and screening mechanism

In the minimal closure (19) with minimally coupled matter, test bodies follow metric geodesics, so there is no additional *direct* fifth force from a matter–scalar coupling. Any observable deviation from GR arises indirectly through the scalar contribution to the metric sourced by $T^{\text{info}}_{\mu\nu}$.
Accordingly, the operative "screening" mechanism in this setup is the massive/pinned regime of Section F.4: if m_χ is large compared to the inverse length scales probed by an experiment, scalar gradients and their stress-energy are exponentially suppressed outside the source region, making exterior observables indistinguishable from GR within stated precision. Chameleon-like screening requires an environment-dependent effective potential (typically via explicit matter coupling), which is not assumed in the minimal CAP-II closure and would constitute an additional modeling layer.

F.6 Binary-pulsar and radiative constraints (qualitative placement)

Binary-pulsar tests constrain departures from GR through strong-field dynamics and radiative channels. In scalar–tensor theories with matter coupling, light scalars typically generate dipole radiation and are therefore tightly constrained; see [31]. CAP-II in its minimal form does not introduce a direct matter–scalar coupling, so the leading dipole-radiation mechanism of Jordan-frame scalar–tensor theories is not automatically present. Operationally, the relevant CAP-II constraint is again that the information sector remain short-ranged or sufficiently weak in the exterior/radiative zone so that waveform phase evolution and timing observables are consistent with the GR templates used in the analysis.

G Wigner–Smith interface details: basis invariance, loss models, and calibration

This appendix complements Section 9 by recording standard identities and practical interface points needed for quantitative use of Wigner–Smith delay as a calibrated proxy for routing overhead. The emphasis is operational: which quantities are basis-independent, how to handle non-unitarity/loss at the interface level, and how to define calibration rules that make lapse ratios testable.

G.1 Trace identities and a density-of-states reading (unitary case)

Let $S(E) \in U(M)$ be the on-shell scattering matrix for M asymptotic channels and define the Wigner–Smith operator

$$Q(E) := -\mathrm{i}\,\hbar\, S(E)^\dagger \,\frac{\mathrm{d}S}{\mathrm{d}E}(E). \tag{92}$$

Since $S^\dagger S = \mathbf{1}$, one has $Q(E) = Q(E)^\dagger$. The scalar delay used in the main text is

$$\tau_{\text{WS}}(E) := \frac{1}{M}\mathrm{Tr}Q(E). \tag{93}$$

Determinant identity. Using $\frac{\mathrm{d}}{\mathrm{d}E}\log\det S = \mathrm{Tr}\Big(S^{-1}\frac{\mathrm{d}S}{\mathrm{d}E}\Big)$ and unitarity ($S^{-1} = S^{\dagger}$), one obtains

$$\mathrm{Tr}Q(E) = -\mathrm{i}\,\hbar\,\frac{\mathrm{d}}{\mathrm{d}E}\log\det S(E). \tag{94}$$

Equivalently, writing $\det S(E) = \mathrm{e}^{\mathrm{i}\Theta(E)}$ with total scattering phase $\Theta(E) \in \mathbb{R}$,

$$\mathrm{Tr}Q(E) = \hbar\,\frac{\mathrm{d}\Theta}{\mathrm{d}E}(E). \tag{95}$$

Density-of-states interface. In standard scattering theory, the energy derivative of the total scattering phase is proportional to the (excess) density of states in the interaction region (Friedel/Krein-type relations). At the interface level, (94) can therefore be read as: the WS trace measures a spectral delay/dwell observable which is naturally interpreted as a density-of-states proxy at the chosen probe energy. For background and conventions, see [21–24].

G.2 Channel-basis dependence and a basis-invariant scalar observable

Let $U \in U(M)$ be an energy-independent change of channel basis and set $S'(E) := U\,S(E)\,U^{\dagger}$. Then

$$Q'(E) = -\mathrm{i}\,\hbar\,S'(E)^{\dagger}\,\frac{\mathrm{d}S'}{\mathrm{d}E}(E) = U\,Q(E)\,U^{\dagger}, \tag{96}$$

so the trace and eigenvalues of $Q(E)$ are basis-invariant:

$$\mathrm{Tr}Q'(E) = \mathrm{Tr}Q(E), \qquad \mathrm{Spec}(Q'(E)) = \mathrm{Spec}(Q(E)). \tag{97}$$

This is the main reason to prefer $\mathrm{Tr}Q(E)$ (or $M^{-1}\mathrm{Tr}Q(E)$) as a scalar interface observable: it is insensitive to static unitary mixing of measurement ports.

If the effective basis transformation is energy-dependent, $U = U(E)$, then $\mathrm{Tr}Q(E)$ acquires an additive contribution from the apparatus dispersion. Operationally, CAP-II treats this as part of the calibration: the same probe configuration and derivative regularization are used throughout the dataset and a reference region is used to normalize out apparatus-dependent offsets (Section 9.4).

Non-reciprocity. Non-reciprocal devices generally have scattering matrices that are not symmetric ($S \neq S^{\mathsf{T}}$) even in the lossless case. This does not obstruct the WS interface: for unitary $S(E)$, $Q(E)$ remains Hermitian and $\mathrm{Tr}Q(E)$ remains basis-invariant in the sense above. Non-reciprocity therefore affects the detailed channel-resolved delay structure (eigenvectors/eigenphases), but not the scalar trace observable used for calibrated κ-ratio targets.

G.3 Loss, non-unitarity, and a calibrated ratio protocol

Realistic devices can exhibit absorption, dissipation, inelasticity, or imperfect calibration, so the measured $S(E)$ may be non-unitary. CAP-II uses two complementary interface viewpoints.

Hidden-channel completion. Non-unitarity can be modeled by coupling the interaction region to additional unobserved channels (loss ports, absorptive baths). In the enlarged channel space the full scattering matrix $\widetilde{S}(E)$ is unitary, and the WS operator $\widetilde{Q}(E)$ and trace identities above apply to $\widetilde{S}$. The measured $S(E)$ is then a sub-block of $\widetilde{S}(E)$, and $\mathrm{Tr}Q(E)$ is interpreted as a partial delay observable.

Self-energy and renormalized resonance parameters. In device-level models (Section 9.1), coupling to leads/baths induces an energy-dependent self-energy. Equivalently, one writes an effective non-Hermitian Hamiltonian $H_{\text{eff}}(E) = H + \Delta(E) - \mathrm{i}\,\Gamma(E)/2$, so that both the resonance center E_0 and linewidth γ are *renormalized* by the environment. The WS interface does not require a microscopic subtraction of these effects: $E_0(x)$ and $\gamma(x)$ are defined operationally by the measured/simulated $S(E)$ in the chosen probe configuration, and the ratio protocol below uses these renormalized parameters consistently across locations. For background on effective-Hamiltonian and self-energy formulations, see [21, 22].

Ratio protocol (calibration robustness). For lapse tests CAP-II primarily uses *ratios* of calibrated delays (or linewidths) between regions. Fix a probe convention and a reference region x_0. Define the dimensionless overhead proxy by

$$\kappa_{\mathrm{WS}}(x) := \frac{\tau_{\mathrm{WS}}(x)}{\tau_0}, \qquad \kappa_0 := \kappa_{\mathrm{WS}}(x_0), \tag{98}$$

where $\tau_{\mathrm{WS}}(x)$ is evaluated at the local probe energy $E_0(x)$ under the same derivative regularization rule. Then the ratio

$$\frac{\kappa(x)}{\kappa_0} \approx \frac{\kappa_{\mathrm{WS}}(x)}{\kappa_{\mathrm{WS}}(x_0)} = \frac{\tau_{\mathrm{WS}}(x)}{\tau_{\mathrm{WS}}(x_0)} \tag{99}$$

is insensitive to any multiplicative apparatus factor that is common to the two measurements (e.g. an overall port-normalization convention or a global tick-to-second calibration). When the interface uses linewidth extraction in the single-resonance regime, the corresponding ratio form is

$$\frac{\kappa(x)}{\kappa_0} \approx \frac{\gamma(x_0)}{\gamma(x)}, \tag{100}$$

consistent with Section 9.3.

Minimal reporting standard. To make the WS interface auditable, a quantitative report should include:

- the measured (or simulated) $S(E)$ data source and port normalization convention;
- a unitarity diagnostic (e.g. $\|S^\dagger S - \mathbf{1}\|$ over the band) and a stated loss model when non-unitarity is present;
- the derivative regularization choice (grid step, smoothing window, or phase-unwrapping method);
- a stability check under moderate changes of the derivative resolution.

This is the minimal data required to evaluate how sensitive inferred κ ratios are to loss and to apparatus dispersion.

References

[1] Haobo Ma. Computational action principle: Least-discrepancy dynamics and field unification in hpa–omega, 2025. Manuscript in the `docs/papers` repository.

[2] Haobo Ma. The holographic hilbert universe: Constructive spacetime, computational-lapse gravity, and a conditional riemann critical-line rigidity theorem in hpa–omega, 2025. Manuscript in the `docs/papers` repository.

[3] F. T. Leighton, Bruce M. Maggs, and Satish B. Rao. Packet routing and job-shop scheduling in O(Congestion + Dilation) steps. *Combinatorica*, 14(2):167–186, 1994.

[4] F. Thomson Leighton. *Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypercubes*. Morgan Kaufmann, 1992.

[5] Lawrence C. Evans. *Partial Differential Equations*. American Mathematical Society, 2 edition, 2010.

[6] B. Roy Frieden. *Physics from Fisher Information: A Unification*. Cambridge University Press, 1998.

[7] Marcel Reginatto. Derivation of the schrödinger equation from the principle of minimum fisher information. *Physical Review A*, 58(3):1775–1778, 1998.

[8] James W. York. Role of conformal three-geometry in the dynamics of gravitation. *Physical Review Letters*, 28(16):1082–1085, 1972.

[9] G. W. Gibbons and S. W. Hawking. Action integrals and partition functions in quantum gravity. *Physical Review D*, 15(10):2752–2756, 1977.

[10] David Lovelock. The einstein tensor and its generalizations. *Journal of Mathematical Physics*, 12(3):498–501, 1971.

[11] John F. Donoghue. General relativity as an effective field theory: The leading quantum corrections. *Physical Review D*, 50(6):3874–3888, 1994.

[12] C. P. Burgess. Quantum gravity in everyday life: General relativity as an effective field theory. *Living Reviews in Relativity*, 7(1):5, 2004.

[13] I. Z. Fisher. Scalar mesostatic field with regard for gravitational effects. *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, 18:636–640, 1948. English translation: arXiv and collected translations; classical massless scalar solution.

[14] A. I. Janis, E. T. Newman, and J. Winicour. Reality of the schwarzschild singularity. *Physical Review Letters*, 20(16):878–880, 1968.

[15] Richard Arnowitt, Stanley Deser, and Charles W. Misner. The dynamics of general relativity. In Louis Witten, editor, *Gravitation: An Introduction to Current Research*, pages 227–265. Wiley, 1962.

[16] Eric Gourgoulhon. 3+1 formalism in general relativity: Bases of numerical relativity. *Living Reviews in Relativity*, 10(1):1, 2007.

[17] M. A. Naimark. *Spectral Functions of a Symmetric Operator*. Izdat. Akad. Nauk SSSR, 1940. Russian; Naimark dilation theorem source.

[18] W. F. Stinespring. Positive functions on c^*-algebras. *Proceedings of the American Mathematical Society*, 6(2):211–216, 1955.

[19] Karl Kraus. *States, Effects, and Operations: Fundamental Notions of Quantum Theory*. Springer, 1983.

[20] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, 2000.

[21] Claude Mahaux and Hans A. Weidenmüller. *Shell-Model Approach to Nuclear Reactions*. North-Holland, 1969.

[22] Supriyo Datta. *Electronic Transport in Mesoscopic Systems*. Cambridge University Press, 1995.

[23] Eugene P. Wigner. Lower limit for the energy derivative of the scattering phase shift. *Physical Review*, 98(1):145–147, 1955.

[24] F. T. Smith. Lifetime matrix in collision theory. *Physical Review*, 118(1):349–356, 1960.

[25] Gregory Breit and Eugene Wigner. Capture of slow neutrons. *Physical Review*, 49(7):519–531, 1936.

[26] Robert M. Wald. *General Relativity.* University of Chicago Press, 1984.

[27] Sean M. Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* Addison-Wesley, 2004.

[28] Randall J. LeVeque. *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems.* Society for Industrial and Applied Mathematics, 2007.

[29] Carl Brans and Robert H. Dicke. Mach's principle and a relativistic theory of gravitation. *Physical Review*, 124(3):925–935, 1961.

[30] Clifford M. Will. *Theory and Experiment in Gravitational Physics.* Cambridge University Press, Cambridge, 2 edition, 1993.

[31] Clifford M. Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(4), 2014.

[32] Irwin I. Shapiro. Fourth test of general relativity. *Physical Review Letters*, 13(26):789–791, 1964.