Computational Action Principle: Least-Discrepancy Dynamics and Field Unification in HPA–Ω

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January 1, 2026

Abstract

We propose and formalize a *Computational Action Principle* (CAP) in the unified Holographic Polar Arithmetic (HPA) and Ω-theory framework [1–3]. The guiding thesis is operational: physical "laws" are not externally imposed rules but minimal-cost error-correction and steady-state constraints required for finite-resolution readout to remain self-consistent over long horizons. The core tension is layered. At the ontic layer, the universe is modeled by a continuous unitary scan and local quantum updates (Weyl pairs, PQCA). At the readout layer, observers access only discrete projections with finite information capacity, inducing canonical tick structures (Ostrowski/Zeckendorf) and structural mismatch measured by discrepancy.

CAP upgrades this mismatch into a variational principle: dynamics selects configurations minimizing a *least-discrepancy* functional subject to local covariance and implementation constraints. Under standard closure assumptions (locality, diffeomorphism invariance, and second-order metric equations), the macroscopic gravitational field equation is forced to be the Einstein equation (with a cosmological constant term) by Lovelock-type uniqueness; discrepancy and implementation costs enter only through an effective stress tensor and potential.

We further motivate an Ω action in which the Fisher-information amplitude of an information density field is minimally coupled to gravity, and the *routing overhead* of compiling local updates to nearest-neighbor circuits appears as a computational lapse field, providing an operational interpretation of gravitational time dilation. Gauge fields arise as compensating connections for local phase readout errors implied by Weyl complementarity, while matter is modeled as topologically locked phase defects.

Finally, we provide reproducible toy experiments: numerical star-discrepancy comparisons across scan slopes illustrate the finite-depth optimality of the golden branch, and an FFT-based Poisson solver verifies that a localized defect source produces an approximate $1/r$ phase potential and an inverse-square "phase pressure" acceleration in the near-field regime.

Keywords: Holographic Polar Arithmetic (HPA); Ω theory; Computational Action Principle; least discrepancy; Fisher information; Weyl pair; routing overhead; emergent gravity; gauge connection; topological defect.

Conventions. Unless otherwise stated, log denotes the natural logarithm. "mod 1" refers to reduction in $\mathbb{R}/\mathbb{Z}$. We use the mostly-plus metric signature $(-,+,+,+)$ and set $c = 1$ in theoretical derivations unless explicitly restored.

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1 Introduction: from "laws" to variationalized error-correction

Interpreting physical laws as *error-correction algorithms* of a holographic computational system is scientifically meaningful only if the claim can be written as a closed variational principle. The Ω framework starts from a fixed global state ω_Ω: ontically, the universe is a static ray/state, and what observers call "time" and "dynamics" arise from finite readout and implementation constraints [2, 3]. This viewpoint is supported by a small set of axioms (O1–O6) emphasizing finite information capacity (holographic bounds) [4–6], causal locality, and an approximate isometric bulk–boundary encoding consistent with operator-algebraic quantum error correction and entanglement wedge reconstruction [7–9].

Within the HPA–Ω semantics, an observer faces an irreducible *structural mismatch* between two layers:

- **Ontic layer (Layer 0):** continuous unitary scan (irrational rotation / Weyl algebra) and local unitary updates (PQCA, quantum walks).
- **Readout layer (Layer 2):** finite-resolution projections (windows), discrete ticks generated by canonical coding (Ostrowski/Zeckendorf), and bounded information capacity.

Readout discretization is not optional: it is an operational consequence of finite information axioms. Therefore, "laws" should be understood as the *minimal-cost constraints* required for the continuous ontic evolution and the discrete readout statistics to remain mutually consistent over long horizons. The goal of this paper is to formalize this constraint as a variational principle that

- yields closed field equations at the macroscopic level,
- identifies a minimal action functional encoding readout mismatch and implementation cost,
- and supports toy-model numerical verification of key intermediate links.

Outline. Section 2 states the minimal axioms and the scan–projection readout semantics. Section 3 introduces discrepancy as a quantitative mismatch measure and shows how a phase potential sourced by coarse-grained discrepancy reproduces the Newtonian limit for isolated defects. Section 4 formulates CAP and motivates the Ω action as a minimal coupling between gravity, Fisher information, and routing overhead. Sections 5–7 derive field equations and interpret gravity and gauge fields operationally. Sections 8–9 discuss matter as topological defects and the variational role of the golden branch. Section 10 provides reproducible toy experiments, and Section 11 discusses testable consequences and model boundaries.

2 Axiomatic frame and readout semantics: minimal structure in HPA–Ω

2.1 Ω axioms (O1–O6) and the "no external time" starting point

We adopt the following axioms as a minimal commitment [2, 3]:

- **O1 (Ω axiom).** The theory is specified by a single global state ω_Ω (not an ensemble); external time is not fundamental.
- **O2 (finite information / holographic bound).** The distinguishable dimension of any finite region is bounded by an exponential of its boundary area (holographic scaling) [4–6].

- **O3 (causal locality).** Local algebras remain finitely propagating under discrete-step evolution; correlation functions are determined by ω_Ω and a sequence of local automorphisms. In lattice systems this is compatible with Lieb–Robinson-type finite-velocity bounds [10].
- **O4 (holographic map / QEC structure).** The bulk–boundary map is approximately isometric on a code subspace and supports entanglement-wedge reconstruction (operator-algebraic QEC semantics) [7–9].
- **O5 (scan–projection readout).** Observer "time" arises from a scan orbit combined with finite-resolution projection; readout probabilities are given by an effective state ρ_{eff} and a POVM/effects family $E_k^{(\varepsilon)}$ [11,12].
- **O6 (Weyl scan algebra).** Scan shift and phase multiplication form a Weyl pair (U, V) satisfying

$$UV = e^{2\pi i\alpha} VU, \qquad \alpha \notin \mathbb{Q}, \tag{1}$$

implying intrinsic complementarity (non-simultaneous diagonalizability); see e.g. the irrational rotation algebra literature [13].

These axioms can be read as a minimal operational contract. O1 fixes the ontic "whole" (a single state) and removes external time as a primitive. O2 makes resolution a physical constraint rather than an analyst's convenience. O3–O4 provide the structural inputs needed for a local-to-global story: locality in the algebraic evolution and a bulk–boundary encoding consistent with QEC semantics. O5 states that the observer's timeline is not the ontic time but a *scan–projection composite*; O6 fixes the irreducible complementarity of this composite via Weyl noncommutativity.

Operationally, these axioms force any readout to confront a triad:

$$\text{projection} \ \wedge \ \text{complementarity} \ \wedge \ \text{finite information}. \tag{2}$$

CAP will be formulated as a minimal variational closure of the mismatch induced by this triad.

2.2 Readout operators: irrational rotation, windows, and mechanical words

In the minimal model, the scan is an irrational rotation on the circle:

$$x_k = x_0 + k\alpha \pmod 1, \qquad \alpha \notin \mathbb{Q}, \quad x_0 \in [0,1). \tag{3}$$

For irrational α, the orbit is equidistributed mod 1 (Weyl's theorem) [14], providing the canonical "uniform ontic measure" that discrepancy quantifies at finite readout depth. Given a binary window $W \subset [0,1)$ (e.g. an interval), the readout sequence is the mechanical word

$$s_k = \mathbf{1}_W(x_k) \in \{0,1\}. \tag{4}$$

For an irrational rotation, binary partitions generate Sturmian flows. The resulting symbolic dynamics has minimal complexity (exactly $n+1$ distinct length-n factors), making it the canonical "least structured" non-periodic readout stream [15,16]. The golden branch yields the Fibonacci word and equips the tick index with canonical Ostrowski numeration, degenerating to Zeckendorf decomposition in the golden case [1,17]. In CAP terms, canonical coding is not an aesthetic choice but an operational compression: it is how a finite observer assigns stable addresses to scan events under limited capacity.

Readout statistics at finite resolution ε are specified in operational quantum language by a POVM/effects family $\{E_k^{(\varepsilon)}\}_k$ with $\sum_k E_k^{(\varepsilon)} = \mathbb{1}$ and

$$P_k^{(\varepsilon)} = \mathrm{Tr}\Big(\rho_{\text{eff}}\, E_k^{(\varepsilon)}\Big). \tag{5}$$

This keeps "discrete observed events" inside standard operational semantics while making explicit the dependence on finite resolution. In particular, the same ontic state ω_Ω can induce different effective dynamics at different ε through the coarse-grained instrument $\{E_k^{(\varepsilon)}\}$, which is precisely the layer separation CAP exploits.

2.3 Orbit trace and finite part: a canonical regularization convention

Long-time averages in the scan–readout setting are not arbitrary: CAP requires a fixed convention for taking regulated limits from discrete orbits to continuum functionals. We adopt a canonical *orbit trace* / Abel finite-part convention (Convention R1) [1, 3, 18].

Conceptually, regulated readout sums often differ by additive constants that depend on the chosen regularization scheme. In CAP this ambiguity is not ignorable: mismatch is promoted to a source term, and source terms shift physical potentials unless the constant is fixed consistently. Convention R1 fixes the additive constants in a way compatible with the scan semantics (Abel-type regularization along the orbit), ensuring that "discrepancy accumulation" admits a well-defined coarse-grained continuum interpretation that can be coupled to covariant field equations.

3 Least discrepancy: from mismatch accumulation to phase potential and the Newtonian limit

3.1 Discrepancy as a readout mismatch measure

For a point set $\{x_n\}_{n=1}^N \subset [0,1)$, define the one-dimensional *star discrepancy* [19]

$$D_N^* := \sup_{a\in[0,1]} \left| \frac{1}{N} \sum_{n=1}^{N} \mathbf{1}_{[0,a)}(x_n) - a \right|, \qquad E_N := N D_N^*. \tag{6}$$

In the scan model $x_n = x_0 + n\alpha \pmod 1$, E_N is interpreted as the accumulated mismatch between finite-prefix readout statistics and the uniform ontic measure. CAP treats mismatch as *structural*: it cannot be eliminated by longer observation but can be controlled.

To connect D_N^* to scan readout stability, write the orbit as $x_n = x_0 + n\alpha \pmod 1$ and consider interval-window counts

$$s_n = \mathbf{1}_W(x_0 + n\alpha), \qquad S_N(W) = \sum_{n=0}^{N-1} s_n, \qquad W \subset [0,1) \text{ an interval.} \tag{7}$$

For $W = [0,a)$ one has $S_N(W) = \sum_{n=1}^N \mathbf{1}_{[0,a)}(x_n)$ and therefore

$$E_N = \sup_{a\in[0,1]} |S_N([0,a)) - Na| . \tag{8}$$

Thus, a uniform interval-count bound immediately yields a star-discrepancy bound.

Continued fractions and Ostrowski expansion. Let $\alpha = [0; a_1, a_2, \dots] \in (0,1) \setminus \mathbb{Q}$ and let $(q_k)_{k\geq 0}$ be the convergent denominators defined by

$$q_{-1} = 0, \quad q_0 = 1, \quad q_{k+1} = a_{k+1} q_k + q_{k-1}. \tag{9}$$

Every $N \in \mathbb{N}_{>0}$ admits a unique Ostrowski expansion relative to α,

$$N = \sum_{k=0}^{m} b_k q_k, \tag{10}$$

with digits satisfying $0 \leq b_0 < a_1$, $0 \leq b_k \leq a_{k+1}$ for $k \geq 1$ and the standard admissibility constraint (if $b_k = a_{k+1}$ then $b_{k-1} = 0$); see e.g. [16, 20].

Theorem 3.1 (Ostrowski–Denjoy–Koksma bound for interval counts)**.** *Let $\alpha = [0; a_1, a_2, \dots] \in (0,1) \setminus \mathbb{Q}$ and let q_k be the convergent denominators. Let $W \subset [0,1)$ be any interval and let $S_N(W) = \sum_{n=0}^{N-1} \mathbf{1}_W(x_0 + n\alpha)$. For every $N \in \mathbb{N}_{>0}$ write the Ostrowski expansion $N = \sum_{k=0}^{m} b_k q_k$. Then for every $x_0 \in [0,1)$,*

$$|S_N(W) - N\,\mu(W)| \le 2\sum_{k=0}^{m} b_k \le 2\sum_{k=0}^{m} a_{k+1}, \tag{11}$$

where μ is Lebesgue measure on $[0,1)$.

Proof. Let $f := \mathbf{1}_W - \mu(W)$. Then f has bounded variation $\mathrm{Var}(f) = 2$ and $\int_0^1 f\,\mathrm{d}\mu = 0$. For each $k \ge 0$, the Denjoy–Koksma inequality for rotations gives a uniform bound at convergent times:

$$\left|\sum_{j=0}^{q_k-1} f(x_0 + j\alpha)\right| \le \mathrm{Var}(f) = 2, \tag{12}$$

see e.g. [21–23]. Now expand $N = \sum_{k=0}^{m} b_k q_k$ and decompose the length-N sum into b_m blocks of length q_m, then b_{m-1} blocks of length q_{m-1}, and so on (the standard Ostrowski block decomposition). Applying the q_k-time bound to each block yields

$$\left|\sum_{n=0}^{N-1} f(x_0 + n\alpha)\right| \le 2\sum_{k=0}^{m} b_k. \tag{13}$$

Since $\sum_{n=0}^{N-1} f(x_0 + n\alpha) = S_N(W) - N\mu(W)$, the first inequality follows. The second inequality follows from the digit bounds $b_0 < a_1$ and $b_k \le a_{k+1}$ for $k \ge 1$. ☐

Corollary 3.2 (Golden optimality for a discrepancy proxy)**.** *Fix a depth $m \ge 0$ and define the continued-fraction proxy*

$$C_m(\alpha) := \sum_{k=0}^{m} a_{k+1}, \qquad \alpha = [0; a_1, a_2, \dots]. \tag{14}$$

Then $C_m(\alpha) \ge m+1$ for every irrational α, with equality if and only if $a_i = 1$ for all $1 \le i \le m+1$ (the golden branch prefix). In particular, the golden slope $\alpha = \varphi^{-1} = [0; 1, 1, 1, \dots]$ uniquely minimizes the upper bound in Theorem 3.1 at every finite depth.

Proof. Since each $a_i \in \mathbb{N}$, one has $a_i \ge 1$ and therefore $C_m(\alpha) \ge m+1$, with equality if and only if $a_i = 1$ for $1 \le i \le m+1$. The uniqueness of continued fractions yields the last statement. ☐

Remark 3.3 (Star discrepancy bound and logarithmic growth for bounded type)**.** *Applying Theorem 3.1 to $W = [0, a)$ and taking the supremum over a yields*

$$E_N = ND_N^* \le 2\sum_{k=0}^{m} a_{k+1}. \tag{15}$$

If α is of bounded type, i.e. $\sup_k a_k \le A < \infty$, then $E_N \le 2A(m+1)$. Moreover, since $a_{k+1} \ge 1$ implies $q_{k+1} \ge q_k + q_{k-1}$, one has $q_k \ge F_k$ (Fibonacci), hence $m = O(\log N)$ whenever $q_m \le N < q_{m+1}$. Therefore $E_N = O(\log N)$ for bounded type slopes.

This quantitative control should be contrasted with resonance-prone slopes that admit exceptionally good rational approximations on intermediate scales. In CAP semantics, good rational approximation corresponds to *phase locking* over finite depth: the scan visits window boundaries in structured patterns that amplify readout mismatch before equidistribution asserts itself. Least discrepancy is therefore not only a statement about asymptotic equidistribution but a statement about finite-resolution stability under repeated projection.

3.2 Finite-depth optimality of the golden branch

The previous subsection provides a concrete finite-depth proxy $C_m(\alpha) = \sum_{k=0}^{m} a_{k+1}$ controlling mismatch via Denjoy–Koksma and Ostrowski decomposition. The golden slope

$$\alpha_\varphi := \varphi^{-1} = \frac{\sqrt{5}-1}{2}, \qquad [0; 1, 1, 1, \dots], \tag{16}$$

minimizes this proxy uniformly at every depth (Corollary 3.2) and is the archetype of “most badly approximable” numbers [24].

Canonical ticks via Ostrowski truncation. At finite depth m, an observer effectively accesses only an m-truncated Ostrowski expansion of scan indices; this induces a discrete tick structure whose stability depends on the continued-fraction prefix [16,20]. In the golden branch, the truncation specializes to Zeckendorf/Fibonacci addressing, supplying a canonical “clock” without external tuning [17].
In CAP terms, the golden branch is not aesthetic but variational: for a fixed readout resolution, it minimizes the *worst-case* upper bound controlling mismatch accumulation, thereby lowering the cost required for long-term self-consistent readout. Pointwise E_N at a fixed N and x_0 can still fluctuate among bounded-type slopes (Section 10); CAP's claim is the uniform stability of the bound that governs sustainable readout.

3.3 Phase potential and phase pressure: mismatch sources generate $1/r$ fields

Omega Dynamics lifts coarse-grained mismatch into a continuum source [25]. Let $\sigma(x)$ be a mismatch density obtained from a regulated continuum limit of E_N (Convention R1). We define the *phase potential* Φ as the stationary point of the quadratic functional $S_\Phi[\Phi; \sigma]$ in (19); equivalently, Φ solves the Poisson equation

$$\Delta\Phi = 4\pi\rho_\Phi, \qquad \rho_\Phi = \kappa_\Phi\,\sigma. \tag{17}$$

We then define the *phase pressure* vector field

$$\mathbf{P}_\Phi := -\nabla\Phi. \tag{18}$$

The Poisson form is the natural weak-field, slow-variation limit: in the Newtonian limit of GR one obtains $\Delta\phi = 4\pi G\rho$ for the gravitational potential ϕ [26,27]. CAP uses the same operator as the minimal continuum lift of mismatch accumulation when promoted to an effective potential.

Variational closure for the phase potential. Independently of GR, the Poisson equation is the Euler–Lagrange equation of the quadratic field functional

$$S_\Phi[\Phi; \sigma] := \int_{\mathbb{R}^3} \mathrm{d}^3x \left(\frac{1}{8\pi} |\nabla\Phi|^2 + \kappa_\Phi\,\sigma\,\Phi \right), \tag{19}$$

with fixed source σ and appropriate decay/boundary conditions. Varying $\Phi \mapsto \Phi + \epsilon\delta\Phi$ and integrating by parts yields

$$\delta S_\Phi = \int \mathrm{d}^3x\,\delta\Phi \left(-\frac{1}{4\pi}\Delta\Phi + \kappa_\Phi\sigma \right), \qquad \Rightarrow \qquad \Delta\Phi = 4\pi\kappa_\Phi\sigma. \tag{20}$$

This recovers the Poisson-type sourcing used above. CAP therefore treats the Poisson lift not as an extra postulate but as the *least-action* closure for a scalar potential field that encodes mismatch cost at the macroscopic level.

Exterior solution and mismatch charge. Let σ be compactly supported in $\mathbb{R}^3$ and assume $\Phi \to 0$ at spatial infinity. Using the Green function identity $\Delta(1/|\mathbf{x}|) = -4\pi\delta(\mathbf{x})$ [28], the unique decaying solution of

$$\Delta\Phi = 4\pi\kappa_\Phi\sigma \tag{21}$$

is

$$\Phi(\mathbf{x}) = -\kappa_\Phi \int_{\mathbb{R}^3} \frac{\sigma(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\,\mathrm{d}^3\mathbf{y}. \tag{22}$$

In particular, for $r = |\mathbf{x}| \to \infty$ one has the multipole expansion

$$\Phi(r) = -\frac{M}{r} + O(r^{-2}), \qquad M := \kappa_\Phi \int_{\mathbb{R}^3} \sigma(\mathbf{y})\,\mathrm{d}^3\mathbf{y}. \tag{23}$$

Thus the monopole coefficient M is fixed by the total mismatch charge $\int \sigma$ and the calibration constant κ_Φ.

Remark 3.4 (Signed discrepancy vs. mismatch charge)**.** *The empirical measure* $\mu_N = \frac{1}{N}\sum_{n=1}^{N}\delta_{x_n}$ *and the uniform measure* μ *satisfy* $\int(\mu_N - \mu) = 0$*, so a* signed *equidistribution defect would have zero total charge and would not generate a monopole* $1/r$ *term. CAP's* σ *is therefore not the signed measure* $\mu_N - \mu$*; it is a* nonnegative coarse-grained cost density *associated with sustaining readout consistency (and, in the Newtonian closure of Appendix E, it can be tied to an effective energy density). This is precisely why an isolated defect sector can carry nonzero total mismatch charge and produce a monopole potential.*

Remark 3.5 (Flux characterization)**.** *By the divergence theorem and* $\Delta\Phi = 4\pi\kappa_\Phi\sigma$*, one has*

$$\int_{S_R} \mathbf{P}_\Phi \cdot \mathrm{d}\mathbf{S} = \int_{B_R} \nabla\cdot\mathbf{P}_\Phi\,\mathrm{d}^3\mathbf{x} = -\int_{B_R} \Delta\Phi\,\mathrm{d}^3\mathbf{x} = -4\pi\kappa_\Phi \int_{B_R} \sigma\,\mathrm{d}^3\mathbf{x}, \tag{24}$$

so in the limit $R \to \infty$ *the monopole strength satisfies*

$$M = -\frac{1}{4\pi}\lim_{R\to\infty}\int_{S_R} \mathbf{P}_\Phi \cdot \mathrm{d}\mathbf{S}. \tag{25}$$

This Gauss-law relation parallels the curvature-flux characterization of topological sectors in Section 8.

For an isolated localized source (compactly supported σ), the exterior solution is harmonic; in the far field the monopole term dominates, and for an approximately spherically symmetric core one may write

$$\Phi(r) = -\frac{M}{r}, \qquad \mathbf{a}(r) \equiv \mathbf{P}_\Phi(r) = -\frac{M}{r^2}\,\hat{\mathbf{r}}, \tag{26}$$

recovering the Newtonian inverse-square form as a mismatch-induced phase-pressure limit. This establishes CAP's first closed chain:

$$\text{discrete readout mismatch (discrepancy)} \;\Rightarrow\; \text{phase potential } \Phi \;\Rightarrow\; \text{phase pressure} \;\Rightarrow\; \text{Newtonian limit.} \tag{27}$$

4 Computational Action Principle and the Ω action

4.1 Two notions of "time" and the geometry of implementation cost

HPA defines *scan time* as the iteration count of a unitary scan. Ω theory introduces a second operational notion: *implementation time*. Even if ontic evolution is unitary and reversible, an observer can only implement and read out local updates through finite hardware constraints. In the Ω program, microscopic evolution is modeled as a partitioned quantum cellular automaton

(PQCA), and a local PQCA step on a finite region is exactly compilable into a one-dimensional nearest-neighbor circuit [2]. This construction is compatible with standard QCA locality results (e.g. unitarity + causality ⇒ localizability) [29–31]. The required circuit depth defines a *routing overhead* field $\kappa(x)$, i.e. the implementation cost of moving information through the local network to realize the update. Appendix F records model-independent circuit-theoretic bounds supporting κ as an operational invariant.

This naturally induces a computational lapse:

$$\mathcal{N}(x) = \frac{\kappa_0}{\kappa(x)}, \qquad \mathrm{d}\tau_{\mathrm{loc}}(x) = \mathcal{N}(x)\,\mathrm{d}t, \qquad \kappa(x) \geq \kappa_0. \tag{28}$$

Intuitively: higher routing overhead means fewer effective local logical updates per unit background depth $\mathrm{d}t$, and therefore a slower local clock. This is the computational interpretation of gravitational time dilation: local clocks slow down because implementing local dynamics consumes more routing budget.

The dictionary becomes concrete in static GR. If we identify $\mathcal{N}(x)$ with the GR lapse, then in Schwarzschild coordinates [26, 32]

$$\mathcal{N}(r) = \sqrt{1 - \frac{2GM}{rc^2}}, \qquad \frac{\kappa(r)}{\kappa_0} = \frac{1}{\mathcal{N}(r)} = 1 + \frac{GM}{rc^2} + O(r^{-2}), \tag{29}$$

so gravitational redshift is operationally equivalent to computational slowdown in the implementation layer.

4.2 Why a Fisher term: information geometry as a minimal covariant quadratic form

To lift readout statistics into a covariant action, we require a local, coordinate-invariant, quadratic form that measures infinitesimal statistical distinguishability of an effective density field. Under standard information-geometric axioms (monotonicity under coarse-graining, locality, and scale naturalness), the Fisher metric is the canonical choice [33, 34]. In particular, Fisher information is the unique (up to scale) Riemannian metric on probability manifolds that is contractive under stochastic maps (coarse-graining), matching the operational fact that finite-resolution readout cannot increase distinguishability.

Remark 4.1 (Uniqueness input (Čencov))**.** *On finite probability simplices, Čencov's theorem states that, up to an overall constant factor, the Fisher information metric is the unique Riemannian metric that is monotone under Markov morphisms (stochastic coarse-grainings) [33, 34]. CAP uses this uniqueness as the quantitative reason why a Fisher-gradient term is the minimal canonical quadratic penalty compatible with finite-resolution readout semantics.*

Let $\varrho(x) \geq 0$ be an information density and define the Fisher functional in covariant form:

$$I_F[\varrho] = \int \mathrm{d}^4x\,\sqrt{-g}\; g^{\mu\nu}\frac{\nabla_\mu\varrho\,\nabla_\nu\varrho}{\varrho} = 4\int \mathrm{d}^4x\,\sqrt{-g}\; g^{\mu\nu}(\nabla_\mu\chi)(\nabla_\nu\chi), \qquad \chi \equiv \sqrt{\varrho}. \tag{30}$$

This suggests a minimal scalar degree of freedom χ (the Fisher amplitude) coupled to gravity.

4.3 Minimal Ω action: Einstein–Hilbert + Fisher amplitude

We take the minimal Ω action to be

$$S_\Omega = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{R-2\Lambda}{16\pi G} - \lambda_F\, g^{\mu\nu}(\nabla_\mu\chi)(\nabla_\nu\chi) - V(\chi^2) + \mathcal{L}_m\right], \qquad \varrho = \chi^2. \tag{31}$$

Here $V(\chi^2)$ is an effective potential encoding readout-stability and topological-sector costs, and $\mathcal{L}_m$ collects additional matter/gauge degrees of freedom. CAP interprets (31) as a minimal-cost closure consistent with O1–O6 and with the requirement that mismatch and implementation overhead must be expressible as local covariant functionals.

Interpretation. The Einstein–Hilbert term fixes the macroscopic geometry under the closure assumptions of Section 5.3. The Fisher-amplitude term is the minimal local quadratic penalty for spatial/temporal variation in distinguishability (readout information density). The potential V encodes the "quantized penalties" required to stabilize discrete readout sectors and to maintain nontrivial topological sectors. The matter/gauge sector captures compensating degrees of freedom required for local consistency under phase complementarity (Section 7).

4.4 Cost–density identification: from routing overhead to information density

To inject implementation cost into the continuum action, we identify a dimensionless cost field

$$s(x) \equiv \frac{\kappa(x)}{\kappa_0} = \frac{1}{\mathcal{N}(x)} \geq 1, \tag{32}$$

and relate it to the information density by a smooth local map. In the weak-field, slow-variation regime, a leading-order model is

$$\varrho(x) = \varrho_0\, s(x)^p, \tag{33}$$

or more generally $\varrho = \varrho_0 F(s)$ with F twice differentiable near $s = 1$, in which case the dominant term reduces to a power law in the relevant asymptotic expansion. Then the Fisher-gradient term becomes a routing-gradient energy: $\chi = \sqrt{\varrho} \propto s^{p/2}$.

Remark 4.2 (Why power laws appear at leading order)**.** *If* $\varrho = \varrho_0 F(s)$ *with* $F(1) = 1$ *and* F *positive and* C^2 *near* $s = 1$*, then* $\log \varrho = \log \varrho_0 + \log F(s)$ *admits a Taylor expansion in* $\log s$*:*

$$\log \varrho = \log \varrho_0 + p \log s + O\Big((\log s)^2\Big), \qquad p := \frac{\mathrm{d}\log F}{\mathrm{d}\log s}\bigg|_{s=1}. \tag{34}$$

Exponentiating yields $\varrho = \varrho_0 s^p\,(1 + O((\log s)^2))$*, justifying* (33) *as the universal leading-order form.*

This provides CAP's second chain: implementation overhead κ controls $s = 1/\mathcal{N}$; s controls ϱ; gradients of ϱ generate Fisher energy and therefore gravitational backreaction through the information stress tensor.

4.5 CAP as a variational principle: embedding least discrepancy into the potential

CAP asserts that among all effective field configurations $(g_{\mu\nu}, \chi, \psi_m, \dots)$ compatible with the Ω axioms and boundary data, physical configurations extremize the total action (31), with the readout mismatch encoded as a least-discrepancy penalty in the potential:

$$V(\chi^2) = V_{\text{disc}}(\chi^2) + V_{\text{topo}}(\chi^2) + V_0. \tag{35}$$

Here V_{disc} encodes quantized penalties induced by window projection and discrepancy accumulation (stabilizing discrete readout sectors), while V_{topo} encodes the maintenance cost of topological sectors (defects, winding). Including gauge symmetry adds compensating connections and Yang–Mills-type terms in $\mathcal{L}_m$ without changing the minimal gravity–Fisher backbone.

CAP in one sentence. *Among all covariant effective configurations compatible with finite-resolution readout, the realized configuration is the stationary point that minimizes accumulated readout discrepancy and implementation overhead in a single action.*

5 Closed field equations: from variation to Einstein gravity with information stress

5.1 Metric variation: information stress tensor and Einstein equation

Varying the action (31) with respect to the metric yields the Einstein equation sourced by matter plus an *information stress tensor*:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \left(T^{(m)}_{\mu\nu} + T^{\text{info}}_{\mu\nu} \right), \tag{36}$$

where

$$T^{\text{info}}_{\mu\nu} = 2\lambda_F \left(\nabla_\mu \chi \, \nabla_\nu \chi - \frac{1}{2} g_{\mu\nu} (\nabla\chi)^2 \right) - g_{\mu\nu} V(\chi^2), \qquad (\nabla\chi)^2 \equiv g^{\rho\sigma} \nabla_\rho \chi \, \nabla_\sigma \chi. \tag{37}$$

Thus, once the Fisher-amplitude sector is accepted as the minimal covariant quadratic form encoding readout distinguishability, the tensorial structure of macroscopic gravity is fixed; mismatch and implementation costs enter through $T^{\text{info}}_{\mu\nu}$ and the model-dependent $V(\chi^2)$.

5.2 Information-field variation: amplitude equation and conserved flow

Varying with respect to χ gives

$$2\lambda_F \, \Box\chi - \frac{\mathrm{d}V}{\mathrm{d}\chi} = 0, \qquad \frac{\mathrm{d}V}{\mathrm{d}\chi} = 2\chi \, \frac{\mathrm{d}V}{\mathrm{d}\varrho}, \qquad \varrho = \chi^2. \tag{38}$$

By diffeomorphism invariance, the total stress tensor is covariantly conserved. When the χ equation holds, the information sector can be treated as separately conserved within the effective theory.

More explicitly, the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ implies

$$\nabla^\mu \left(T^{(m)}_{\mu\nu} + T^{\text{info}}_{\mu\nu} \right) = 0. \tag{39}$$

If the matter sector is conserved (or if matter exchanges flow with the χ sector through explicit couplings), then the χ equation ensures consistency of the split. This is the covariant statement that least-discrepancy closure must respect local conservation laws imposed by diffeomorphism invariance.

It is useful to define a natural "information current":

$$J_\mu \equiv \chi \nabla_\mu \chi = \frac{1}{2} \nabla_\mu \varrho, \tag{40}$$

which captures the simplest covariant flow associated with the density gradient.

5.3 Covariant closure and uniqueness: why the macroscopic equation must be Einstein

CAP emphasizes that the macroscopic field equation should not be postulated by hand. If the effective continuum limit satisfies:

- locality and diffeomorphism invariance,
- metric field equations of at most second differential order,
- and asymptotic flatness (or a controlled cosmological background),

then in four dimensions the only symmetric, divergence-free rank-2 tensor built from the metric and its derivatives up to second order is $G_{\mu\nu} + \Lambda g_{\mu\nu}$, up to an overall coupling [35]. (The next Lovelock density is the Gauss–Bonnet term, which is topological in 4D and does not contribute to local metric field equations.) Therefore the gravitational skeleton is uniquely Einsteinian; CAP's micro-to-macro content lies in the structure of the effective stress tensor (mismatch and cost) and in the sector dictionary that links readout/implementation objects to covariant fields.

This is the central closure mechanism behind CAP: *once the effective description is required to be local, covariant, and second-order, there is no freedom left in the left-hand side of the metric equation.* All microstructural choices—scan slope selection, coding stability, discrepancy penalties, compilation overhead—must appear on the right-hand side, as effective sources and potentials.

6 Gravity as computation: refractive index, routing cost, and Fermat principle

6.1 Computational refractive index and propagation delay

The computational lapse $\mathcal{N}(x) = \kappa_0/\kappa(x)$ can be interpreted as an effective slowdown factor: under a fixed background depth $\mathrm{d}t$, the number of realizable local logical updates is reduced by routing overhead. This is equivalent to viewing spacetime as a *computational refractive medium* with index

$$n_{\mathrm{comp}}(x) \sim \frac{1}{\mathcal{N}(x)} = \frac{\kappa(x)}{\kappa_0}. \tag{41}$$

Regions of larger n_{comp} incur larger effective delays, matching the operational content of gravitational time delay (e.g. Shapiro delay) when the dictionary $\mathcal{N} \leftrightarrow \sqrt{-g_{00}}$ is adopted.

In the weak-field regime, one may write $g_{00} \approx -(1 + 2\phi)$ with Newtonian potential ϕ, so $\mathcal{N} \approx 1 + \phi$ [26,27]. Then $n_{\mathrm{comp}} \approx 1 - \phi$, making the optical-length language directly compatible with standard lensing and delay calculations.

6.2 A computational form of Fermat's principle

In this readout–implementation language, "geodesics" are not introduced as primitive geometric minimizers; rather, propagation follows trajectories minimizing *total implementation cost.* In a slowly varying background, this reduces to Fermat's principle in the effective index $n_{\mathrm{comp}}(x)$: light and information follow paths of stationary optical length, which in turn reproduces the familiar macroscopic bending and delay phenomena.

Concretely, for spatial paths γ connecting two endpoints, CAP suggests an effective optical functional

$$\mathcal{T}[\gamma] \propto \int_\gamma n_{\mathrm{comp}}(\mathbf{x})\, \mathrm{d}\ell, \tag{42}$$

so that stationary $\delta\mathcal{T} = 0$ yields the Euler–Lagrange equations equivalent to null geodesics of the associated optical metric. The novelty is semantic rather than algebraic: n_{comp} is not a material refractive index but an operational proxy for routing overhead.

CAP thus supplies a second closed chain:

$$\text{routing overhead } \kappa \ \Rightarrow\ \text{computational lapse } \mathcal{N} \ \Rightarrow\ \text{effective metric component } g_{00} \ \Rightarrow\ \text{refraction/delay and e} \tag{43}$$

7 Emergent gauge fields: compensating local phase errors under Weyl complementarity

7.1 Weyl-pair complementarity and unavoidable local phase jitter

In the scan–phase algebra, the Weyl relation

$$UV = e^{2\pi i\alpha} VU \tag{44}$$

implies that scan localization (readout tick/position) and phase-mode localization (frequency/energy) cannot be simultaneously sharpened. In CAP language, finite-resolution readout of phase necessarily introduces local phase errors $\delta\theta(x)$, which can accumulate into macroscopic structure.

This is the field-theoretic translation of the layer tension. The readout layer cannot simultaneously enforce sharp time ticks (scan localization) and sharp phase coherence (mode localization) because the underlying operators do not commute. Therefore, any effective continuum description that treats phase as a local degree of freedom must include a mechanism that tracks and compensates unavoidable phase jitter.

7.2 From local rephasing to a connection field

If readout errors are interpreted as uncontrolled local rephasings

$$\psi(x) \mapsto e^{i\lambda(x)}\psi(x), \tag{45}$$

then keeping the effective action invariant under this redundancy requires a compensating connection A_μ and the replacement of derivatives by covariant derivatives:

$$\nabla_\mu \;\to\; D_\mu = \nabla_\mu - iqA_\mu, \qquad A_\mu \mapsto A_\mu + \partial_\mu\lambda. \tag{46}$$

This is the standard minimal-coupling construction for local $U(1)$ redundancy [36, 37]. At second order in derivatives, the minimal local gauge kinetic term is the Yang–Mills quadratic form $F_{\mu\nu}F^{\mu\nu}$ [38,39], which can be included in $\mathcal{L}_m$ in (31).

Proposition 7.1 (Quadratic gauge rigidity)**.** *Assume an effective local action contains a gauge sector for a $U(1)$ connection A_μ that is (i) invariant under $A_\mu \mapsto A_\mu + \partial_\mu\lambda$, (ii) local and Lorentz covariant, (iii) built using at most first derivatives of A_μ, and (iv) quadratic in A_μ and its derivatives in the bulk. Then, up to an overall normalization and boundary terms, the unique bulk quadratic kinetic term is $F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.*

Proof. Gauge invariance forbids dependence on A_μ without derivatives in the pure gauge sector, and the only gauge-invariant tensor built from first derivatives of A_μ is the antisymmetric combination $F_{\mu\nu}$. Any local Lorentz scalar quadratic in first derivatives therefore reduces (up to integration by parts) to a linear combination of $F_{\mu\nu}F^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$; the latter is a total derivative in four dimensions and does not affect the bulk Euler–Lagrange equations [36,37]. □

Nonabelian extension. If the effective phase redundancy is promoted from a $U(1)$ rephasing to a local internal symmetry group G, the same compensating logic yields a nonabelian connection $A_\mu = A^a_\mu T^a$ and curvature $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ [37]. CAP does not fix G at the present level; it fixes that some compensating connection is required once local phase alignment is demanded.

The key point is structural: gauge fields need not be postulated as external symmetries; they arise as the minimal compensating structure demanded by *local phase consistency* under Weyl complementarity and finite readout resolution. In this sense, gauge dynamics is the variationally cheapest "patch" that keeps local phase descriptions compatible with finite-resolution readout.

8 Origin of matter: topologically locked phase defects and inertial budget

8.1 Topological charge from curvature flux (Dirac/Wu–Yang)

CAP treats stable localized "matter" as a defect sector of the compensating connection introduced in Section 7. The relevant rigidity statement is standard in gauge theory and topology: nontrivial defect sectors are characterized by integral cohomology classes (Chern numbers) extracted from curvature flux [37, 39–41].

U(1) example. Let $U = \mathbb{R}^3 \setminus \{0\}$ be space with an isolated puncture. A $U(1)$ gauge field is described by local 1-forms A with curvature 2-form $F = \mathrm{d}A$, which satisfies the Bianchi identity $\mathrm{d}F = 0$. Surround the defect by a sphere $S^2 \subset U$. Global consistency of phase (single-valued transition functions between local gauges) implies the first Chern number is an integer:

$$Q_{\mathrm{top}} := \frac{1}{2\pi} \int_{S^2} F \in \mathbb{Z}. \tag{47}$$

Lemma 8.1 ("Closed but not exact" on a punctured region)**.** *On $U = \mathbb{R}^3 \setminus \{0\}$, a smooth 2-form F with $\mathrm{d}F = 0$ and $\int_{S^2} F = 2\pi n \neq 0$ cannot be exact. Equivalently, no global gauge choice can remove the defect sector.*

Proof. If $F = \mathrm{d}A$ globally on U, then Stokes' theorem gives $\int_{S^2} F = \int_{S^2} \mathrm{d}A = 0$, contradicting $\int_{S^2} F = 2\pi n \neq 0$. In de Rham language, the flux labels the nontrivial class in $H^2(U) \cong H^2(S^2) \cong \mathbb{Z}$ [37]. □

8.2 Mass as sustained budget: internal winding and geometric impedance

Omega Dynamics further proposes an organizing hypothesis: stable matter corresponds to phase errors that cannot be removed by local gauge redefinitions (closed but not exact), i.e. *topologically locked defects.* Maintaining a nontrivial topological sector requires persistent implementation budget, which manifests macroscopically as inertial mass and as resistance to propagation (geometric impedance).

In HPA language, such defects can be modeled as localized impedance centers in the scan–phase dynamics: routing around a local algebraic obstruction requires extra scan cycles and accumulates additional geometric phase. From the readout perspective, this appears as propagation delay and curvature, consistent with the computational-lapse dictionary of Section 4.1.

A quantitative interface inequality. The phase-potential closure (Section 3.3) assigns to a nonnegative coarse-grained mismatch cost density σ a monopole strength

$$M := \kappa_\Phi \int_{\mathbb{R}^3} \sigma(\mathbf{x})\, \mathrm{d}^3\mathbf{x}, \tag{48}$$

so that $\Phi(r) \sim -M/r$ for isolated sources. A topologically nontrivial gauge sector (Lemma 8.1) necessarily carries nonzero curvature and therefore incurs a positive maintenance cost in any local quadratic effective action. In soliton models this appears as Bogomol'nyi-type bounds that lower-bound energy by topological charge [42, 43]. CAP packages the same structural fact as a minimal, testable interface inequality:

Proposition 8.2 (Topological lower bound on mismatch charge)**.** *Assume sustaining a defect sector with $|Q_{\mathrm{top}}|$ requires mismatch charge $Q_\sigma := \int_{\mathbb{R}^3} \sigma\, \mathrm{d}^3x$ satisfying*

$$Q_\sigma \geq m_0\, |Q_{\mathrm{top}}| \tag{49}$$

for some microscopic per-charge budget $m_0 > 0$. Then the exterior monopole coefficient obeys

$$M \geq \kappa_\Phi\, m_0\, |Q_{\mathrm{top}}|. \tag{50}$$

At the current stage, m_0 and the full defect spectrum (charges, stability, interactions) must be computed from explicit microscopic scan/readout architectures; this remains an open task within the broader Ω program [2].

9 Variational inevitability of the golden branch: from minimal discrepancy to canonical clocks

The distinguished role of the golden ratio in HPA–Ω follows from two independent mechanisms:

1. **Discrepancy minimization.** At finite readout depth, the golden branch minimizes monotone discrepancy proxies controlled by continued-fraction coefficients (Section 3.2). This reduces mismatch accumulation and therefore the total cost required to sustain self-consistent readout.

2. **Shortest canonical coding.** In the golden branch, Ostrowski numeration degenerates to Zeckendorf decomposition: every integer has a unique representation as a sum of non-consecutive Fibonacci numbers [17]. This yields a canonical discrete tick structure that naturally decomposes finite budgets across scales without external tuning.

These two mechanisms are logically independent: the first is about *anti-resonance* (hardness of rational approximation) and mismatch suppression, while the second is about *addressability* (canonical normal form for finite tick budgets). Their conjunction explains why the golden branch repeatedly appears as a stable fixed point across the HPA–Ω chain: it simultaneously lowers mismatch cost and shortens the bookkeeping needed to maintain readout coherence.

In CAP terms, the golden branch is not an aesthetic ornament; it is a variational attractor selected by readout sustainability under finite information and Weyl complementarity.

10 Numerical toy experiments: reproducible checks of key CAP links

This section provides two minimal simulations that check intermediate CAP links: (i) discrepancy accumulation across scan slopes; (ii) defect source → Poisson phase potential → approximate $1/r$ field. The corresponding reference implementations are included in Appendix C.

10.1 Experiment A: star discrepancy for Kronecker sequences

We consider the Kronecker orbit

$$x_n = (x_0 + n\alpha) \bmod 1, \qquad x_0 = 0.12345, \tag{51}$$

and compute the one-dimensional star discrepancy D_N^* and the accumulated mismatch $E_N = ND_N^*$.

Interpretation. Theorem 3.1 provides a worst-case upper bound on E_N in terms of the continued-fraction proxy $C_m(\alpha)$, and Corollary 3.2 shows that the golden branch minimizes this proxy at every finite depth. The empirical E_N at a fixed x_0 can still fluctuate among bounded-type slopes; the relevant CAP statement is the uniform stability of the bound that governs sustainable readout.

N	$\alpha = \varphi^{-1}$ (golden)	$\alpha = \sqrt{2} - 1$ (silver)	$\alpha = \mathrm{e} - 2$	$\alpha = \pi - 3$
200	1.790506	1.835317	2.557223	3.672939
500	1.802811	1.741094	1.599947	7.110437
1000	1.752245	1.706770	1.268638	8.140322
2000	1.911260	2.062419	1.973620	17.143745
5000	2.173936	2.258847	2.934050	37.846869
10000	2.942666	2.909670	2.845196	60.373228
20000	2.818185	2.595185	4.146376	61.891876

Table 1: **Accumulated mismatch $E_N = ND_N^*$ for selected scan slopes.** Smaller E_N indicates lower finite-prefix mismatch. Bounded-type slopes (golden/silver) maintain low mismatch over large N, while more resonance-prone slopes can exhibit much larger E_N in the same range.

slope α	A	B	R^2
φ^{-1} (golden)	0.268900	0.109201	0.771640
$\sqrt{2} - 1$ (silver)	0.240277	0.316830	0.753593
$\mathrm{e} - 2$	0.409865	-0.666467	0.490403
$\pi - 3$	14.499032	-83.104486	0.899618

Table 2: **Finite-range log-fit for E_N on the sampled N values.** Bounded-type slopes show modest A at this resolution; slopes with strong intermediate-scale rational approximations can exhibit much larger apparent coefficients in finite ranges.

Log-fit diagnostic. To quantify the "slow growth" trend, we fit a simple finite-range model

$$E_N \approx A \log N + B \tag{52}$$

by least squares on the sampled N values (Table 1). This fit is not a theorem; it is an operational diagnostic consistent with the $E_N = O(\log N)$ bound for bounded-type slopes (Remark after Corollary 3.2).

Dependence on the phase offset x_0. Because D_N^* is a *star* discrepancy (intervals anchored at 0), the measured E_N depends on the phase offset x_0 even for a fixed slope α. To avoid selection bias from a single seed, we report summary statistics over a uniform grid $x_0 \in \{0, \frac{1}{16}, \dots, \frac{15}{16}\}$ at $N = 20000$:

Theorem-bound sanity check (golden/silver). For quadratic irrational slopes with eventually periodic continued fractions (golden and silver), the proxy bound in Theorem 3.1 can be computed explicitly from the convergent denominators. Table 4 reports the corresponding $U_N := 2\sum_{j \le m+1} a_j$ (where $q_m \le N < q_{m+1}$) and the empirical ratio E_N/U_N.

10.2 Experiment B: FFT solution of Poisson equation and an approximate $1/r$ phase potential

We solve the periodic-grid Poisson equation

$$\Delta\Phi = 4\pi\rho \tag{53}$$

in a three-dimensional periodic box using FFT, with ρ a single lattice-site point source. After solving, we compute radial shell averages of Φ about the source center.

For a representative run with $n = 64$ grid, the near-field behavior is well-approximated by $\Phi(r) \sim -\mathrm{const}/r$ (so $r\langle\Phi\rangle$ is approximately constant) before periodic-boundary effects dominate

slope α	min E_N	max E_N	mean	std
φ^{-1} (golden)	2.818185	3.818185	3.073116	0.366927
$\sqrt{2}-1$ (silver)	2.595185	3.595185	2.881189	0.308296
$\mathrm{e}-2$	3.146376	4.893425	3.879091	0.525654
$\pi-3$	37.309031	69.891876	53.662953	10.095011

Table 3: **x_0-sensitivity of E_N at fixed $N = 20000$.** Values are computed over a uniform grid of 16 phase offsets. The worst-case theoretical upper bound from Theorem 3.1 is uniform in x_0; the table reports empirical spread for anchored-interval discrepancy.

	golden $\alpha = \varphi^{-1}$			silver $\alpha = \sqrt{2}-1$		
N	E_N	U_N	E_N/U_N	E_N	U_N	E_N/U_N
200	1.790506	24	0.074604	1.835317	28	0.065547
500	1.802811	28	0.064386	1.741094	32	0.054409
1000	1.752245	32	0.054758	1.706770	36	0.047410
2000	1.911260	34	0.056214	2.062419	36	0.057289
5000	2.173936	38	0.057209	2.258847	40	0.056471
10000	2.942666	40	0.073567	2.909670	44	0.066129
20000	2.818185	44	0.064050	2.595185	48	0.054066

Table 4: **Empirical mismatch vs. theorem proxy upper bound.** Even with anchored-interval star discrepancy and a fixed x_0, the empirical E_N remains well below the uniform proxy bound U_N at these scales.

at larger radii. The script also supports a least-squares fit of the form $\langle\Phi\rangle \approx C_0 - M/r$ over a chosen radius range and reports RMS residuals as a quantitative goodness-of-fit.

$1/r$ fit (representative). Using the values in Table 5 and fitting over $r \in \{2, 3, 4, 5, 6\}$ gives

$$\langle\Phi\rangle(r) \approx C_0 - \frac{M}{r}, \qquad C_0 \approx 0.016236, \quad M \approx 0.854953, \tag{54}$$

with RMS residual $\approx 6.09 \times 10^{-3}$ and $R^2 \approx 0.9964$ on that fit window. This provides a quantitative confirmation of the near-field $1/r$ behavior expected from the Poisson closure.

In the continuum normalization $\Delta\Phi = 4\pi\rho$ with a unit point source $\rho = \delta$, one expects $M = 1$ (Green function normalization). The fitted $M \approx 0.855$ is therefore within $\sim 15\%$ of the continuum value, consistent with finite-grid and periodic-image effects at $n = 64$.

11 Discussion: testable consequences and model boundaries

(1) Time dilation as implementation cost. If an operational estimate of the effective routing overhead $\kappa(x)$ of a region can be extracted (e.g. via compilation-depth proxies inferred from phase delays or scattering-time observables in an explicit microscopic model), then the computational lapse $\mathcal{N} = \kappa_0/\kappa$ predicts redshift and time delay in that region via the dictionary $\mathcal{N} \leftrightarrow \sqrt{-g_{00}}$.

(2) Phenomenology of information stress. Spatially inhomogeneous cost fields $\kappa(x)$ induce Fisher-gradient energy in the χ sector. In weak-field regimes this contributes positively to curvature through $T^{\text{info}}_{\mu\nu}$ and can be comparable to the Newtonian potential in low-acceleration regions, providing a structured space for effective deviations without abandoning the Einsteinian skeleton.

r	$\langle\Phi\rangle$	$r\langle\Phi\rangle$
1	-0.7908	-0.7908
2	-0.4057	-0.8113
3	-0.2775	-0.8326
4	-0.2029	-0.8116
5	-0.1529	-0.7645
6	-0.1195	-0.7172

Table 5: **Representative FFT Poisson output (periodic box).** The near-field shows $r\langle\Phi\rangle \approx$ const, consistent with $\Phi \sim -1/r$.

(3) Topological sectors and dark-matter candidates. Stable defects carrying internal topological charge but neutral under long-range gauge flux could behave as collisionless matter. In CAP, a defect sector labeled by $Q_{\rm top}$ incurs mismatch charge Q_σ and therefore a gravitational monopole strength $M = \kappa_\Phi Q_\sigma$ (Section 3.3). Proposition 8.2 implies the quantitative lower bound $M \geq \kappa_\Phi m_0 |Q_{\rm top}|$. The microscopic per-charge budget m_0 (and the detailed spectrum) remain to be derived from explicit scan/readout architectures or fixed phenomenologically.

Model boundary. This paper closes the minimal backbone from scan–projection readout, discrepancy, and implementation overhead to an Einstein–Fisher action and its field equations. Completing the bridge to Standard Model parameters, a first-principles defect spectrum, and higher-dimensional quasicrystal/Dirac homotopy limits requires additional microscopic input; these are explicit open tasks in the Ω program [2].

12 Conclusion

We formulated a *Computational Action Principle* (CAP) that upgrades the slogan "physical laws as error correction" into a closed variational program within HPA–Ω. The key content is structural:

- discrete readout mismatch is quantified by discrepancy and lifted (under a fixed regularization convention) into continuum source terms;
- under locality, diffeomorphism invariance, and second-order closure, the macroscopic gravitational equation is uniquely Einsteinian (with Λ), while mismatch and implementation costs enter through effective stress and potentials;
- the Ω action provides a minimal backbone coupling gravity to Fisher-information amplitude and to routing overhead (computational lapse), giving an operational interpretation of time dilation as computational slowdown;
- gauge fields arise as compensating connections for local phase readout errors, and matter is modeled as topologically locked defects sustained by implementation budget.

Together, these elements support a unified view: physical structure emerges as the geometric/computational cost required for a finite readout system to remain self-consistent.

A Symbols and objects quick reference

- ω_Ω: global state (Axiom O1).
- α: scan slope for irrational rotation; golden branch $\alpha = \varphi^{-1}$.

- U, V: Weyl pair; $UV = \mathrm{e}^{2\pi\mathrm{i}\alpha} VU$.
- W: readout window; $s_k = \mathbf{1}_W(x_k)$ mechanical word.
- $E_k^{(\varepsilon)}$: readout effects/POVM elements at resolution ε; $P_k^{(\varepsilon)} = \mathrm{Tr}(\rho_{\mathrm{eff}} E_k^{(\varepsilon)})$.
- D_N^*: star discrepancy; $E_N = N D_N^*$ accumulated mismatch.
- $\kappa(x)$: routing overhead (implementation depth); $\mathcal{N}(x) = \kappa_0/\kappa(x)$ computational lapse.
- $\varrho = \chi^2$: information density; χ Fisher-information amplitude.
- Φ: phase potential sourced by mismatch; $\mathbf{P}_\Phi = -\nabla\Phi$ phase pressure.

B Key derivation chains (summary)

B.1 The Ω action and field equations (minimal sector)

The minimal CAP sector is gravity plus Fisher amplitude plus matter:

$$S_\Omega = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G} - \lambda_F\, g^{\mu\nu} (\nabla_\mu \chi)(\nabla_\nu \chi) - V(\chi^2) + \mathcal{L}_m \right]. \tag{55}$$

Metric variation yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{\mathrm{info}} \right), \qquad T_{\mu\nu}^{\mathrm{info}} = 2\lambda_F \left(\nabla_\mu\chi \nabla_\nu\chi - \frac{1}{2} g_{\mu\nu} (\nabla\chi)^2 \right) - g_{\mu\nu} V(\chi^2). \tag{56}$$

Variation in χ yields

$$2\lambda_F \Box\chi - \frac{\mathrm{d}V}{\mathrm{d}\chi} = 0, \qquad \frac{\mathrm{d}V}{\mathrm{d}\chi} = 2\chi \frac{\mathrm{d}V}{\mathrm{d}\varrho}, \quad \varrho = \chi^2. \tag{57}$$

B.2 Second-order covariant closure ⇒ Einstein uniqueness

If the macroscopic continuum limit is local, diffeomorphism invariant, and second-order in the metric, then the divergence-free symmetric rank-2 tensor on the left-hand side is uniquely $G_{\mu\nu} + \Lambda g_{\mu\nu}$ (up to coupling), so the gravitational skeleton must be Einsteinian [35].

B.3 Discrepancy source ⇒ Poisson phase potential ⇒ Newton limit

Theorem 3.1 provides a finite-depth quantitative bound on mismatch accumulation E_N in terms of continued-fraction data. In CAP, a regulated continuum limit of the corresponding mismatch is packaged into a coarse-grained density σ, and one assumes it sources a phase potential:

$$\Delta\Phi = 4\pi\kappa_\Phi \sigma, \qquad \mathbf{P}_\Phi = -\nabla\Phi. \tag{58}$$

For an isolated defect with quantized flux, spherical symmetry enforces the exterior harmonic solution $\Phi = -M/r$, hence $\mathbf{P}_\Phi = -M\hat{\mathbf{r}}/r^2$. Appendix E explains why the Poisson operator is the correct weak-field closure (Newtonian limit of GR) and how the Ω action sources it through an effective energy density.

C Reproducible toy experiments (Python)

This appendix contains reference implementations for Section 10. They require `numpy`. A minimal requirement file is provided in `requirements.txt`.

C.1 Experiment A: star discrepancy and accumulated mismatch E_N

```
import math

import numpy as np


def star_discrepancy_1d(x: np.ndarray) -> float:
    """Compute 1D star discrepancy for points x in [0,1)."""
    x = np.sort(np.asarray(x, dtype=float))
    N = len(x)
    i = np.arange(1, N + 1, dtype=float)
    d1 = np.max(i / N - x)
    d2 = np.max(x - (i - 1) / N)
    return float(max(d1, d2))


def orbit_points(alpha: float, N: int, x0: float = 0.0) -> np.ndarray:
    n = np.arange(N, dtype=float)
    return (x0 + n * alpha) % 1.0


def EN(alpha: float, N: int, x0: float = 0.12345) -> float:
    x = orbit_points(alpha, N, x0=x0)
    D = star_discrepancy_1d(x)
    return float(N * D)


def stats(vals: np.ndarray):
    vals = np.asarray(vals, dtype=float)
    return float(vals.min()), float(vals.max()), float(vals.mean()), float(vals.std())


def continued_fraction(alpha: float, max_terms: int = 64):
    """
    Return partial quotients [a1, a2, ...] for alpha in (0,1) with
    alpha = [0; a1, a2, ...] using floating-point iteration.
    """
    x = float(alpha)
    a = []
    for _ in range(max_terms):
        if x <= 0:
            break
        inv = 1.0 / x
        ai = int(math.floor(inv))
        a.append(ai)
        x = inv - ai
        if abs(x) < 1e-15:
            break
    return a
```

```
def dk_upper_bound(alpha: float, N: int, max_terms: int = 64):
    """
    Denjoy--Koksma/Ostrowski proxy upper bound used in Theorem 03:
      E_N = N D_N^* <= 2 * sum_{j=1}^{m+1} a_j,
    where q_m <= N < q_{m+1} and alpha = [0; a1, a2, ...].
    Returns (U_N, m, C_m) with U_N = 2*C_m and C_m = sum_{j=1}^{m+1} a_j.
    """
    a = continued_fraction(alpha, max_terms=max_terms)
    if not a:
        return float("nan"), -1, float("nan")

    # q_{-1} = 0, q_0 = 1
    q_prev = 0
    q_curr = 1
    m = -1
    for j, aj in enumerate(a, start=1):
        q_next = aj * q_curr + q_prev
        if q_curr <= N < q_next:
            m = j - 1  # current q_curr is q_m
            break
        q_prev, q_curr = q_curr, q_next

    if m < 0:
        # Fallback: N is beyond computed range; treat last available as m.
        m = len(a) - 1

    C_m = float(sum(a[: m + 1]))  # sum_{j=1}^{m+1} a_j
    U_N = 2.0 * C_m
    return U_N, m, C_m


def linfit(xs: np.ndarray, ys: np.ndarray):
    """Least squares fit y = a x + b with R^2."""
    xs = np.asarray(xs, dtype=float)
    ys = np.asarray(ys, dtype=float)
    mx = float(xs.mean())
    my = float(ys.mean())
    sxx = float(((xs - mx) ** 2).sum())
    sxy = float(((xs - mx) * (ys - my)).sum())
    a = sxy / sxx if sxx != 0 else float("nan")
    b = my - a * mx
    sst = float(((ys - my) ** 2).sum())
    sse = float(((ys - (a * xs + b)) ** 2).sum())
    r2 = 1.0 - sse / sst if sst != 0 else float("nan")
    return a, b, r2


if __name__ == "__main__":
    alphas = {
        "golden_phi^-1": (math.sqrt(5) - 1) / 2,
        "sqrt2_minus_1": math.sqrt(2) - 1,
```

```
        "e_minus_2": math.e - 2,
        "pi_minus_3": math.pi - 3,
    }

    Ns = [200, 500, 1000, 2000, 5000, 10000, 20000]

    header = ["N"] + list(alphas.keys())
    print("\t".join(header))
    series = {name: [] for name in alphas}
    for N in Ns:
        row = [str(N)]
        for name, a in alphas.items():
            v = EN(a, N)
            series[name].append(v)
            row.append(f"{v:.6f}")
        print("\t".join(row))

    print("\nFit: E_N ~ A log N + B (least squares on the sampled Ns)")
    xs = np.log(np.asarray(Ns, dtype=float))
    for name, ys_list in series.items():
        ys = np.asarray(ys_list, dtype=float)
        A, B, r2 = linfit(xs, ys)
        print(f"{name}: A={A:.6f}, B={B:.6f}, R2={r2:.6f}")

    # x0 sensitivity (anchored-interval discrepancy depends on phase shift x0).
    N0 = 20000
    x0s = np.arange(16, dtype=float) / 16.0
    print(f"\nPhase-offset sensitivity at N={N0} over x0 in {{0,1/16,...,15/16}}")
    for name, a in alphas.items():
        vals = np.array([EN(a, N0, x0=float(x0)) for x0 in x0s], dtype=float)
        mn, mx, mean, sd = stats(vals)
        print(f"{name}: min={mn:.6f}, max={mx:.6f}, mean={mean:.6f}, std={sd:.6f}")

    # Theorem proxy upper bound (uniform in x0).
    print("\nTheorem proxy upper bound U_N = 2*sum_{j<=m+1} a_j (float CF)")
    for N in Ns:
        print(f"N={N}")
        for name, a in alphas.items():
            U_N, m, C_m = dk_upper_bound(a, N)
            e = EN(a, N)
            ratio = e / U_N if U_N > 0 else float("nan")
            print(f"  {name}: E_N={e:.6f}, U_N={U_N:.6f}, ratio={ratio:.6f}, m={m}, C_m={C_m
```

C.2 Experiment B: 3D periodic Poisson solver via FFT and radial averaging

```
import numpy as np


def poisson_periodic_green_3d(n: int, source_pos=None) -> np.ndarray:
    """
```

```
    Solve Laplacian(phi) = 4*pi*rho on an n x n x n periodic grid (spacing 1) via FFT.
    Gauge: set k=0 mode to 0 (zero-mean potential).
    """
    if source_pos is None:
        source_pos = (n // 2, n // 2, n // 2)

    rho = np.zeros((n, n, n), dtype=float)
    rho[source_pos] = 1.0

    rho_k = np.fft.fftn(rho)

    k = 2 * np.pi * np.fft.fftfreq(n)
    KX, KY, KZ = np.meshgrid(k, k, k, indexing="ij")
    k2 = KX**2 + KY**2 + KZ**2

    phi_k = np.zeros_like(rho_k, dtype=complex)
    mask = k2 != 0
    phi_k[mask] = -4 * np.pi * rho_k[mask] / k2[mask]

    phi = np.real(np.fft.ifftn(phi_k))
    return phi


def radial_average(phi: np.ndarray):
    n = phi.shape[0]
    center = np.array([n // 2, n // 2, n // 2], dtype=float)
    coords = np.indices(phi.shape).reshape(3, -1).T.astype(float)
    r = np.linalg.norm(coords - center, axis=1)
    phi_flat = phi.ravel()

    out = []
    for rad in range(1, n // 4):
        shell = (r >= rad - 0.5) & (r < rad + 0.5)
        if shell.sum() == 0:
            continue
        out.append((rad, float(phi_flat[shell].mean()), float(phi_flat[shell].std()), int(sh
    return out


def fit_inverse_r(stats, r_min: int, r_max: int):
    """
    Fit <Phi>(r) approx C0 - M/r over integer radii r in [r_min, r_max].
    Returns (C0, M, rms).
    """
    rows = [(r, mean) for (r, mean, _std, _cnt) in stats if r_min <= r <= r_max]
    if len(rows) < 2:
        raise ValueError("Need at least two radii for fitting.")

    r = np.asarray([rr for rr, _ in rows], dtype=float)
    y = np.asarray([yy for _, yy in rows], dtype=float)
    x = 1.0 / r
```

```
    # y = a + b x, with b = -M and a = C0
    A = np.vstack([np.ones_like(x), x]).T
    coef, *_ = np.linalg.lstsq(A, y, rcond=None)
    C0, b = float(coef[0]), float(coef[1])
    M = -b
    resid = y - (C0 + b * x)
    rms = float(np.sqrt(np.mean(resid**2)))
    return C0, M, rms


if __name__ == "__main__":
    n = 64
    phi = poisson_periodic_green_3d(n)
    stats = radial_average(phi)

    print("r | <Phi> | r*<Phi> | std | count")
    for r, mean, std, cnt in stats[:12]:
        print(f"{r:2d} | {mean: .6f} | {r*mean: .6f} | {std: .6f} | {cnt}")

    # Example near-field fit window; adjust as needed to avoid periodic-image effects.
    r_min, r_max = 2, 6
    C0, M, rms = fit_inverse_r(stats, r_min=r_min, r_max=r_max)
    print(f"\nFit over r in [{r_min},{r_max}]: <Phi>(r) approx C0 - M/r")
    print(f"C0={C0:.6f}, M={M:.6f}, RMS={rms:.6e}")
```

D Extensible numerical directions (brief)

- **Window-perturbation stability.** Under small perturbations of mechanical-word windows, shifts in empirical statistics can be bounded by Denjoy–Koksma-type estimates, providing a quantitative link between "window noise" and effective defect density.
- **From 1D to higher-dimensional quasicrystals.** The Ω program emphasizes higher-dimensional quasicrystal quantum walks and Dirac homotopy limits as a route to isotropic continuum behavior; numerical diagnostics may use structure factors and phason-strain control.
- **Direct measurement of routing overhead.** In explicit PQCA/circuit models, one can compute local compilation depth $\kappa(x)$ and test the operational dictionary $\mathcal{N} = \kappa_0/\kappa$ against effective redshift, delay, and refraction phenomena.

E Newtonian limit: Poisson equation from Einstein closure and the Ω action

This appendix makes explicit the weak-field chain used in Section 3.3: why a Poisson equation is the correct macroscopic operator in the static, slow-variation limit, and how the Ω action sources it through an effective energy density.

E.1 Poisson equation from the weak-field Einstein equation

In the Newtonian regime (static sources, nonrelativistic velocities, weak fields), write the metric in the standard scalar-potential form

$$ds^2 = -(1+2\phi)\,dt^2 + (1-2\phi)\,d\mathbf{x}^2, \qquad |\phi| \ll 1. \tag{59}$$

To leading order, the 00 component of the Einstein equation yields the Poisson equation

$$\Delta\phi = 4\pi G\,\rho, \tag{60}$$

where $\rho = T_{00}$ is the energy density in the nonrelativistic limit; see e.g. [26,27,32]. In particular, for isolated sources, $\rho = 0$ outside the support of matter and therefore $\Delta\phi = 0$ in the exterior region, forcing $\phi(r) = -GM/r$ under asymptotic flatness.

E.2 Effective energy density from the Fisher-amplitude sector

For the minimal Ω action (31), the information stress tensor is

$$T^{\text{info}}_{\mu\nu} = 2\lambda_F \left(\nabla_\mu\chi\,\nabla_\nu\chi - \frac{1}{2}g_{\mu\nu}(\nabla\chi)^2\right) - g_{\mu\nu}V(\chi^2). \tag{61}$$

In a static weak-field configuration with negligible time derivatives of χ, one may read off an effective energy density

$$\rho_{\text{info}} \equiv T^{\text{info}}_{00} \approx \lambda_F\,(\nabla\chi)^2 + V(\chi^2), \tag{62}$$

up to higher-order corrections in ϕ and in time derivatives. Therefore, the Newtonian potential sourced by the χ sector satisfies

$$\Delta\phi \approx 4\pi G\,(\rho_m + \rho_{\text{info}})\,, \tag{63}$$

which is the precise sense in which mismatch penalties (encoded in V) and implementation-gradient energy (encoded in $(\nabla\chi)^2$ through the cost–density map) backreact on macroscopic geometry.

E.3 Phase potential as a rescaled Newtonian potential

Section 3.3 introduces a phase potential Φ with $\Delta\Phi = 4\pi\rho_\Phi$ and phase pressure $\mathbf{P}_\Phi = -\nabla\Phi$. In the weak-field regime, this is consistent with GR provided one identifies Φ with a rescaled Newtonian potential:

$$\Phi = \gamma\,\phi, \qquad \rho_\Phi = \gamma\,G\,(\rho_m + \rho_{\text{info}})\,, \tag{64}$$

for some calibration constant γ determined by the chosen normalization of Φ and of the mismatch density σ. With this identification, the "phase pressure" acceleration coincides with gravitational acceleration up to the same calibration:

$$\mathbf{a} = -\nabla\phi = -\frac{1}{\gamma}\nabla\Phi = \frac{1}{\gamma}\mathbf{P}_\Phi. \tag{65}$$

E.4 Quantitative fit targets for the toy Poisson experiment

For an isolated defect, the exterior prediction is

$$\Phi(r) \approx C_0 - \frac{M}{r}, \tag{66}$$

with C_0 a gauge constant (zero-mean convention in the periodic FFT solver) and M the effective defect strength. In a periodic box, the far field deviates due to image charges; therefore fits should be performed over an intermediate "near-field" radius range where periodic effects are small. The reference script in Appendix C reports least-squares estimates of (C_0, M) and RMS residuals for a chosen fit window.

F Routing overhead as a circuit-theoretic invariant (basic bounds)

This appendix isolates a minimal, model-independent backbone behind the "routing overhead" κ used in Section 4.1. The goal is not to fix a particular microscopic architecture but to show that (i) a well-defined circuit-depth notion exists, (ii) it obeys universal locality bounds, and (iii) it therefore supports the interpretation of $\mathcal{N} = \kappa_0/\kappa$ as an operational slowdown factor.

F.1 1D nearest-neighbor depth and causal light cones

Consider a one-dimensional chain of qubits (or finite-dimensional local systems) indexed by $i \in \mathbb{Z}$, and a depth-D circuit composed of layers of disjoint nearest-neighbor two-site gates. Let $\mathrm{supp}(O)$ denote the set of sites on which an operator O acts nontrivially.

Lemma F.1 (Light-cone bound for 1D nearest-neighbor circuits)**.** *Let U be a depth-D 1D nearest-neighbor circuit. Then for any local operator O,*

$$\mathrm{diam}\Big(\mathrm{supp}(U^\dagger O U)\Big) \leq \mathrm{diam}(\mathrm{supp}(O)) + 2D. \tag{67}$$

In particular, information (operator support) propagates at speed at most one lattice site per circuit layer.

Proof. Each circuit layer consists of disjoint nearest-neighbor gates. Conjugation by a two-site gate can enlarge the support of an operator by at most one site on each side (because the gate touches at most one new neighbor of the current support). Iterating over D layers yields an expansion by at most D sites to the left and D sites to the right. □

F.2 A universal lower bound on routing overhead

Corollary F.2 (Range lower bound)**.** *Suppose a target unitary U_{tar} maps some strictly local operator at site i to an operator supported at distance at least L away (i.e. $\mathrm{diam}(\mathrm{supp}(U_{\mathrm{tar}}^\dagger O U_{\mathrm{tar}})) - \mathrm{diam}(\mathrm{supp}(O)) \geq 2L$ for some O). Then any 1D nearest-neighbor circuit implementing U_{tar} has depth $\geq L$.*

This is the minimal reason why "routing overhead" is not arbitrary bookkeeping: locality alone enforces depth lower bounds.

F.3 Finiteness and a coarse upper bound

On any finite region of n sites, depth is also finite. A crude universal statement is that any target unitary can be implemented with depth $O(n^2)$ using gate decompositions plus SWAP routing, hence κ is always finite on finite regions [12]. In structured models (e.g. fixed local update rules), one expects much better bounds; CAP does not require the optimal construction, only that $\kappa(x)$ is a well-defined operational cost field.

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